NAME _____ DATE _____

12–6 Solving Problems Involving Quadratic Equations

Objective: To use quadratic equations to solve problems.

Example A landscaper wishes to design a rectangular formal garden that will be 6 m longer than the width. If the area of the garden is to be 135 m^2, find the length and the width.

Solution

Step 1 The problem asks for the length and the width of the garden.

Step 2 Let x = the width in meters.
Then $x + 6$ = the length in meters.

Step 3 Use the formula for the area of a rectangle to write an equation.

$$\text{Length} \times \text{Width} = \text{Area}$$
$$x(x + 6) = 135$$

Step 4 Solve.

$$x^2 + 6x = 135$$
$$x^2 + 6x + 9 = 135 + 9$$
$$(x + 3)^2 = 144$$
$$x + 3 = \pm\sqrt{144}$$
$$x + 3 = \pm 12$$
$$x = -3 \pm 12$$
$$x = 9 \quad \text{or} \quad x = -15$$

Step 5 Disregard the negative root since a negative length has no meaning.

Check: 9: $9(9 + 6) \stackrel{?}{=} 135$

$$9(15) \stackrel{?}{=} 135$$
$$135 = 135\ \surd$$

The width of the garden is 9 m and the length is 15 m.

Solve. Give irrational roots to the nearest tenth. Use your calculator or a table of square roots as necessary.

1. The sum of a number and its square is 72. Find the number.
2. The sum of a number and its square is 30. Find the number.
3. The difference of a number and its square is 110. Find the number.
4. The difference of a number and its square is 132. Find the number.
5. The width of a rectangular garden is 4 m shorter than the length. If the area of the garden is 320 m^2, find the length and the width.
6. An architect wants to design a rectangular building such that the area of the floor is 400 yd^2. The length of the floor is to be 10 yd longer than the width. Find the length and the width of the floor.

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12–6 Solving Problems Involving Quadratic Equations *(continued)*

Solve. Give irrational roots to the nearest tenth. Use your calculator or a table of square roots as necessary.

7. The length of a rectangle is 3 times the width. The area of the rectangle is 48 cm^2. Find the length and the width.

8. The length of a rectangular park is 5 m longer than the width. If the area of the park is 150 m^2, find the length and the width.

9. The length of a rectangle is twice the width. The area of the rectangle is 72 m^2. Find the length and the width.

10. The length of the base of a triangle is twice its altitude. If the area of the triangle is 144 cm^2, find the altitude. (*Hint:* Area of a triangle $= \frac{1}{2} \times$ base $\times$ height.)

11. The altitude of a triangle is 5 m less than its base. The area of the triangle is 80 m^2. Find the base.

12. If the sides of a square are increased by 3 cm, its area becomes 144 cm^2. Find the length of the sides of the original square.

13. If the sides of a square are increased by 4 cm, its area becomes 196 cm^2. Find the length of the sides of the original square.

14. If you square a positive number, add ten times the number and subtract 84, the result is 180. What is the number?

15. Oscar has a rectangular garden that measures 18 m by 15 m. Next year he wants to increase the area to 340 m^2 by increasing the width and the length by the same amount. What will be the dimensions of the garden next year?

16. Working alone, Carrie can paint a house in 3 h less than Walter. Together Carrie and Walter can paint a house in 8 h. How long does it take Walter to paint a house alone?

17. Andrea can ride her bike 2 mi/h faster than Ruby. It takes Andrea 48 min less to travel 50 mi than it does Ruby. What is Andrea's rate in mi/h?

Mixed Review Exercises

Solve.

1. $\frac{\sqrt{n}}{2} = \frac{\sqrt{5}}{1}$

2. $\frac{\sqrt{x-2}}{4} = \frac{3}{2}$

3. $\frac{\sqrt{3t}}{8} = \frac{1}{4}$

4. $\frac{12}{5} = \frac{36}{n}$

5. $m^2 = 12m - 32$

6. $\frac{x-2}{x+3} + x = 2$

In Exercises 7–9, (x_1, y_1) and (x_2, y_2) are ordered pairs of the same direct variation. Find the missing value.

7. $x_1 = 1, y_1 = 3$
   $x_2 = 6, y_2 =$?

8. $x_1 = 8, y_1 =$?
   $x_2 = 4, y_2 = 5$

9. $x_1 = 12, y_1 = 16$
   $x_2 =$?, $y_2 = 8$