Example of the Trapezoidal Rule

>

> *restart*

> *with*(*Student*[*Calculus1*]) :

> *Digits* := 9

$$Digits := 9 \tag{1}$$

Example 2. Use the **Trapezoidal Rule** to approximate $\int_0^3 e^{-x^2}\, dx$ with $N = 3$, 10, and 50 subintervals.

> $f := x \rightarrow e^{-x^2}$

$$f := x \rightarrow e^{-x^2} \tag{2}$$

> $a := 0;\ b := 3$

$$a := 0$$
$$b := 3 \tag{3}$$

>

Approximate the integral by partitioning [0, 3] into <u>3 subintervals</u> of equal width:

> $N := 3$

$$N := 3 \tag{4}$$

> $h := \frac{b-a}{N}$

$$h := 1 \tag{5}$$

> $\frac{h}{2}\left(f(a) + 2\cdot Sum(f(a + i\cdot h), i = 1..N-1) + f(b)\right)$

$$\frac{1}{2} + \sum_{i=1}^{2} e^{-i^2} + \frac{1}{2}\, e^{-9} \tag{6}$$

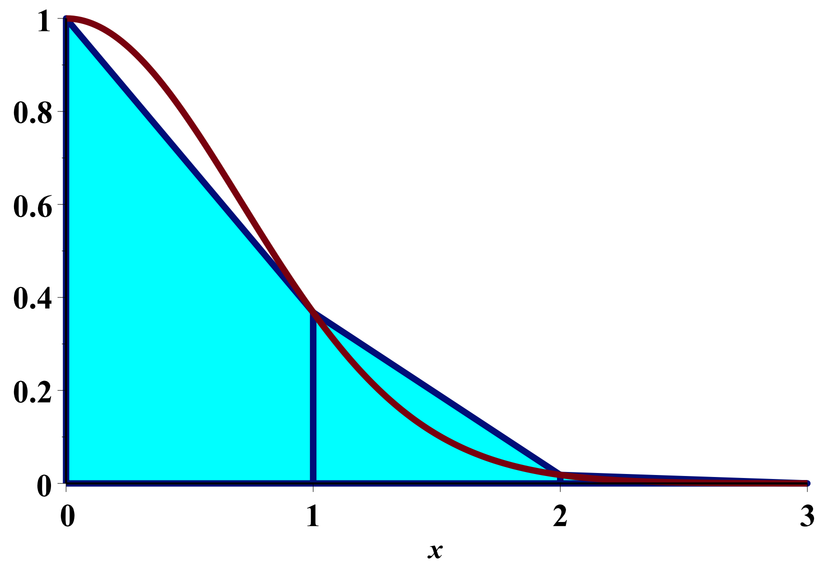
>

> *evalf*(%)

$$0.886256785 \tag{7}$$

>

> $ApproximateInt\Big(f(x), x = a..b, output = plot, partition = N, method = trapezoid, boxoptions = [filled = [color = \text{"Cyan"}, transparency = .9]], tickmarks = \Big(\Big[spacing\Big(\frac{b-a}{N}\Big), default\Big]\Big), thickness = 3, font = [Roman, bold, 12], labelfont = [Roman, 10]\Big)$



An approximation of $\int_0^3 f(x)\,dx$ using trapezoid rule, where $f(x) = e^{-x^2}$ and the partition is uniform. The approximate value of the integral is 0.886256785. Number of subintervals used: 3.

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Approximate the integral by partitioning $[0, 3]$ into <u>10 subintervals</u> of equal width:

> $N := 10$

$$N := 10 \qquad \textbf{(8)}$$

> $h := \frac{b-a}{N}$

$$h := \frac{3}{10} \qquad \textbf{(9)}$$

> $\frac{h}{2}\left(f(a) + 2 \cdot Sum(f(a + i \cdot h), i = 1 .. N - 1) + f(b)\right)$

$$\frac{3}{20} + \frac{3}{10} \sum_{i=1}^{9} e^{-\frac{9}{100} i^2} + \frac{3}{20} e^{-9} \qquad \textbf{(10)}$$

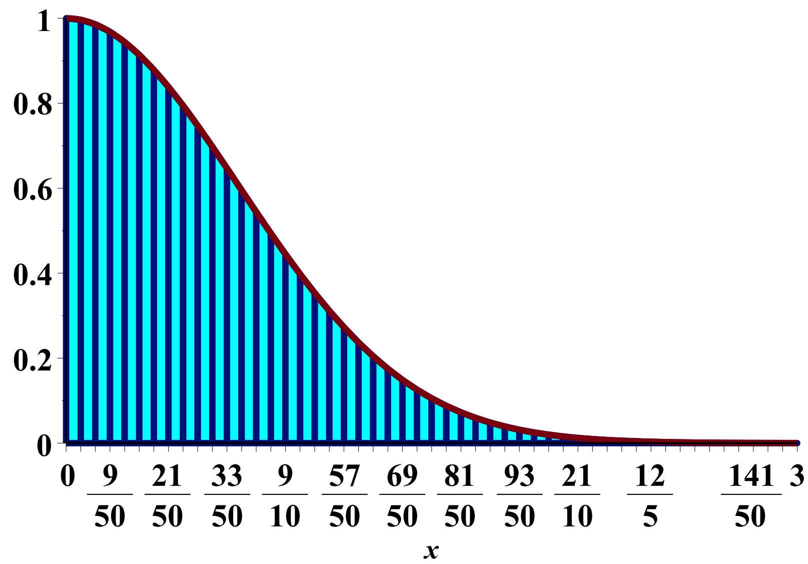
>

> $evalf(\%)$

$$0.886202032 \tag{11}$$

>

> $ApproximateInt\Big(f(x), x = a..b, output = plot, partition = N, method = trapezoid, boxoptions = [filled = [color = \text{"Cyan"}, transparency = .9]], tickmarks = \Big(\Big[spacing\Big(\frac{b-a}{N}\Big), default\Big]\Big), thickness = 3, font = [Roman, bold, 12], labelfont = [Roman, 10]\Big)$



An approximation of $\int_0^3 f(x)\,dx$ using trapezoid rule, where $f(x) = e^{-x^2}$ and the partition is uniform. The approximate value of the integral is 0.886207126. Number of subintervals used: 50.

>

Approximate the integral by partitioning $[0, 3]$ into <u>50 subintervals</u> of equal width:

> $N := 50$

$$N := 50 \tag{12}$$

> $h := \frac{b-a}{N}$

$$h := \frac{3}{50} \tag{13}$$

> $\frac{h}{2}\left(f(a) + 2\cdot Sum(f(a+i\cdot h), i=1..N-1) + f(b)\right)$

$$\frac{3}{100} + \frac{3}{50}\sum_{i=1}^{49} e^{-\frac{9}{2500}i^2} + \frac{3}{100}e^{-9} \tag{14}$$

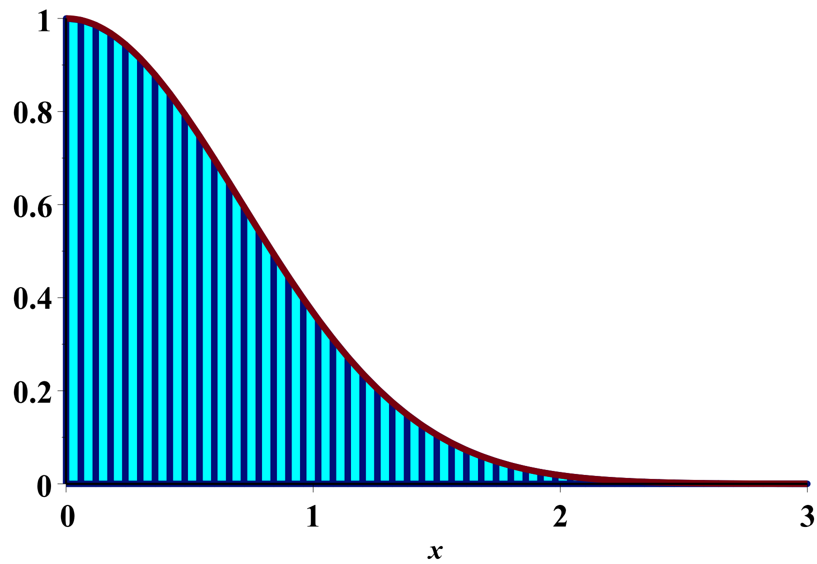
>

> $evalf(\%)$

$$0.886207128 \tag{15}$$

>

> $ApproximateInt(f(x), x = a..b, output = plot, partition = N, method = trapezoid,$
$boxoptions = [filled = [color = \text{"Cyan"}, transparency = .9]], tickmarks$
$= ([spacing(1), default]), thickness = 3, font = [Roman, bold, 12], labelfont$
$= [Roman, 10])$



An approximation of $\int_0^3 f(x)\,dx$ using trapezoid rule, where $f(x) = e^{-x^2}$ and the partition is uniform. The approximate value of the integral is 0.886207126. Number of subintervals used: 50.

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Finally, let us compare the above approximations with the value obtained using the *error function*, which is defined by

$$\mathrm{erf}(x) := \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt.$$

>

Consequently, $\int_0^x e^{-t^2}\,dt = \frac{\sqrt{\pi}}{2}\mathrm{erf}(3)$. Since the error function is built into *Maple*, we have

> $\frac{\sqrt{\pi}}{2}\operatorname{erf}(3)$

$$\frac{1}{2}\sqrt{\pi}\operatorname{erf}(3) \tag{16}$$

> *evalf*(%, 12)

$$0.886207348260 \tag{17}$$

>

**** End ****

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