

[>

## Volumes of Solids of Revolution

[> restart

[> with(Student[Calculus1]) :

[>

**Problem.** Find the volume of a sphere of radius  $r = 2$ .

[>

[>  $f := x \rightarrow \sqrt{4 - x^2}$

$$f := x \rightarrow \sqrt{4 - x^2} \quad (1)$$

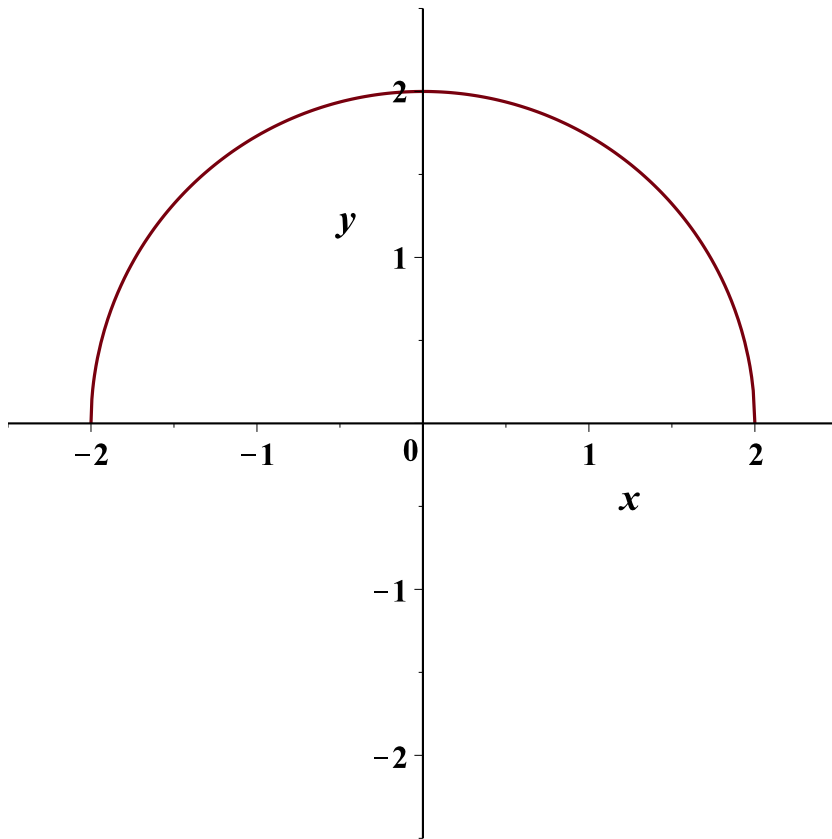
[>

[>  $f(-2), f(0), f(1)$

$$0, 2\sqrt{3} \quad (2)$$

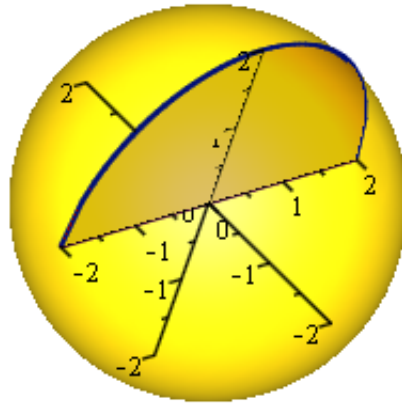
[>

[> plot( $f(x)$ ,  $x = -2..2$ , view = [-2.5..2.5, -2.5..2.5], labels = [x, y], font = [Roman, bold, 12], labelfont = [Roman, 14])



```
=>  
=>
```

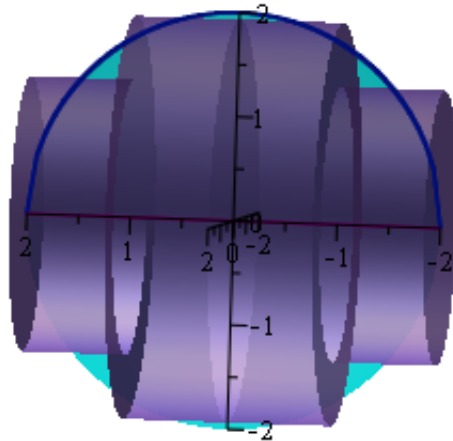
```
VolumeOfRevolution(f(x), x=-2..2, output=plot, showregion=true, orientation=[  
-110, 50, 25], lightmodel=light1)
```



The solid of revolution created on  $-2 \leq x \leq 2$  by rotation of  $f(x) = \sqrt{-x^2 + 4}$  about the axis  $y=0$ . The slice that is rotated is shaded in burgundy.

>

> `VolumeOfRevolution(f(x), x=-2..2, output=plot, orientation=[132, 87, 35],  
lightmodel=light2, method=midpoint, partition=4, showsum);`



The solid of revolution created on  $-2 \leq x \leq 2$  by rotation of

$$f(x) = \sqrt{-x^2 + 4} \text{ about the axis } y = 0.$$

>

> `VolumeOfRevolution(f(x), x=-2..2, output = sum, orientation = [118, 68, 0], method = midpoint, partition = 4);`

$$\pi \left( \sum_{i=1}^4 \left| \left( -\frac{5}{2} + i \right)^2 - 4 \right| \right) \quad (3)$$

> `VolumeOfRevolution(f(x), x=-2..2, output = sumvalue, orientation = [118, 68, 0], method = midpoint, partition = 4)`

$$11 \pi \quad (4)$$

> `evalf(%)`

$$34.55751919 \quad (5)$$

>

> `VolumeOfRevolution(f(x), x=-2..2, output = sum, orientation = [118, 68, 0], method = midpoint, partition = 20);`

$$\frac{1}{5} \pi \left( \sum_{i=1}^{20} \left| \left( -\frac{21}{10} + \frac{1}{5} i \right)^2 - 4 \right| \right) \quad (6)$$

> *VolumeOfRevolution*(*f*(*x*), *x* = -2 ..2, *output* = *sumvalue*, *orientation* = [118, 68, 0],  
*method* = *midpoint*, *partition* = 20)

$$\frac{267}{25} \pi \quad (7)$$

> *evalf*(%)

$$33.55220954 \quad (8)$$

>

> *VolumeOfRevolution*(*f*(*x*), *x* = -2 ..2, *output* = *integral*);

$$\int_{-2}^2 (-\pi (x^2 - 4)) dx \quad (9)$$

$$\int_{-2}^2 \pi \cdot (4 - x^2) dx$$

$$\frac{32}{3} \pi \quad (10)$$

> *evalf*(%)

$$33.51032165 \quad (11)$$

>

>

**Problem 3 on p. 446.** Find the volume of the solid by rotating the region bounded by the curves  $y = \sqrt{x - 1}$ ,  $y = 0$ ,  $x = 5$ , about the  $x$ -axis.

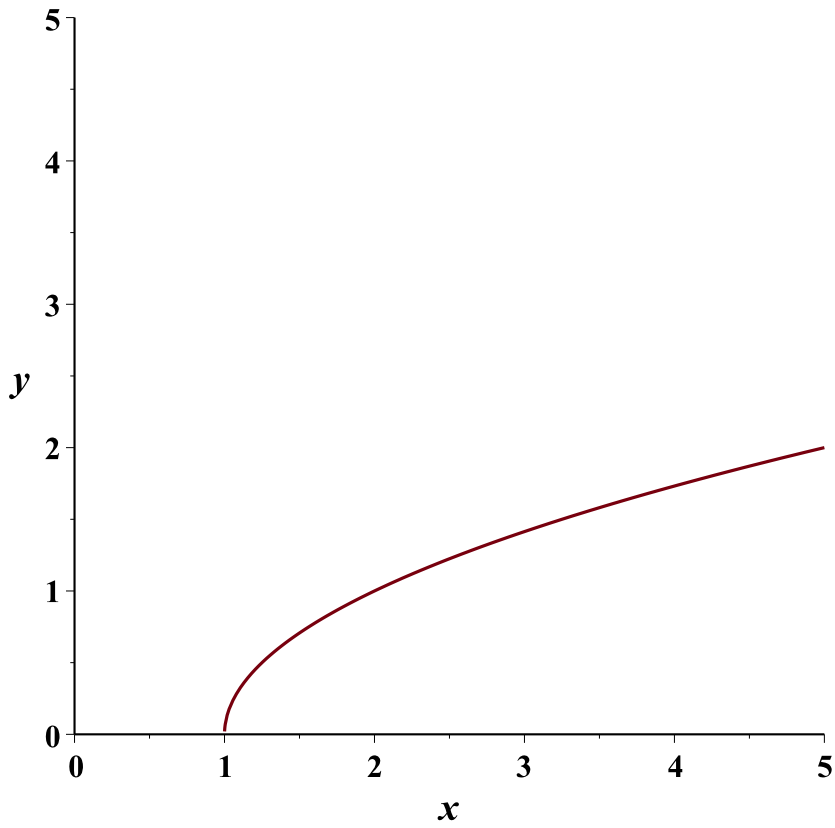
>

> *g* :=  $x \rightarrow \sqrt{x - 1}$

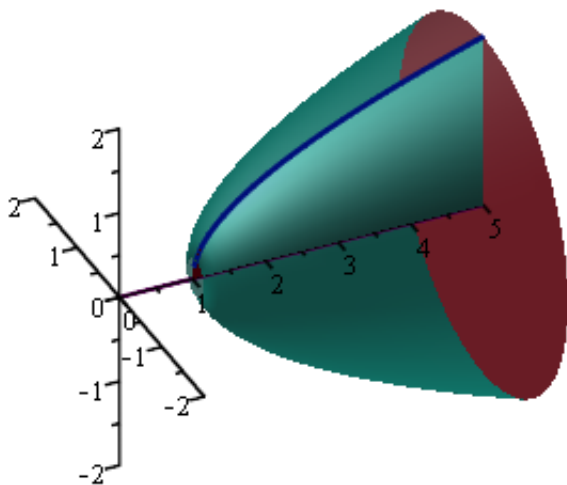
$$g := x \rightarrow \sqrt{x - 1} \quad (12)$$

>

> *plot*(*g*(*x*), *x* = 0 ..5, *view* = [0 ..5, 0 ..5], *labels* = [*x*, *y*], *font* = [*Roman*, *bold*, 12],  
*labelfont* = [*Roman*, 14])



```
>  
> VolumeOfRevolution(g(x), x = 0 .. 5, output = plot, showregion = true, orientation = [  
-115, 57, 0])
```



The solid of revolution created on  $0 \leq x \leq 5$  by rotation of  $f(x) = \sqrt{x-1}$  about the axis  $y=0$ . The slice that is rotated is shaded in burgundy.

```
>
> VolumeOfRevolution(g(x), x = 1 ..5, output = value)
                        8 π
```

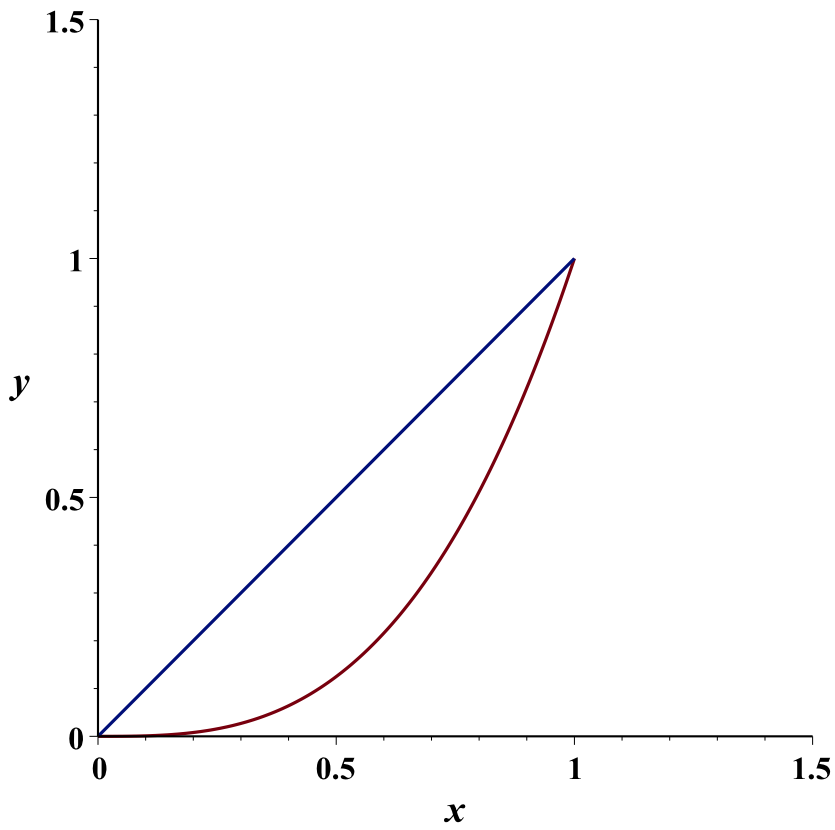
**(13)**

**Problem 7 on p. 446.** Find the volume of the solid by rotating the region bounded by the curves  $y = x^3$ ,  $y = x$ ,  $x \geq 0$ , about the  $x$ -axis.

```
>
> h := x → x3
                        h := x → x3
```

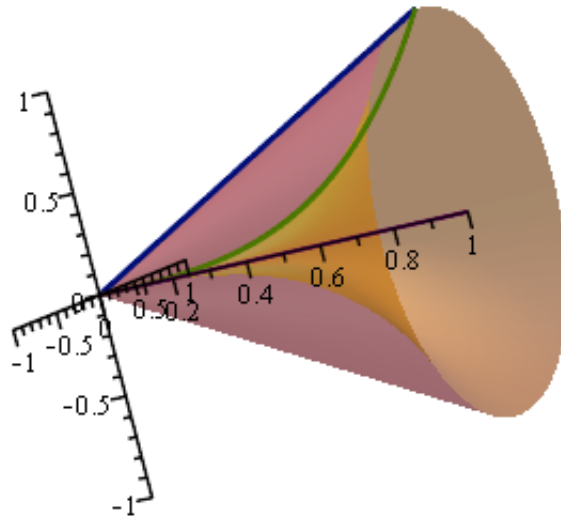
**(14)**

```
> plot([h(x), x], x = 0 ..1, view = [0 ..1.5, 0 ..1.5], labels = [x, y], font = [Roman, bold,
12], labelfont = [Roman, 14])
```



```
>  
> VolumeOfRevolution(x, h(x), x = 0 .. 1, output = plot, orientation = [-140, 75, -75])
```





The solid of revolution created on  $0 \leq x \leq 1$  by rotation of  $f(x) = x$  and  $g(x) = x^3$  about the axis  $y=0$ .

```
==>
```

```
> VolumeOfRevolution(x, h(x), x=0..1, output = value, orientation = [-140, 75, -75])
```

$$\frac{4}{21} \pi$$

**(15)**

```
==>
```

**\*\*\* END \*\*\***

```
>
```