Quiz 6

#6 p.355 Find a general antiderivative of $f(x) = (x-5)^2$.

Soln. Since $f(x) = x^2 - 10x + 25$, $F(x) = \frac{1}{3}x^3 - 5x^2 + 25x + C$.

Or, $F(x) = \frac{1}{3}(x-5)^3 + C$

since $F'(x) = \frac{1}{3}\frac{d}{dx}(x-5)^3 = \frac{1}{3}(3)(x-5)^2 \frac{d}{dx}(x-5) = (x-5)^2$.

#12 p.355 Find a general antiderivative of $f(x) = \sqrt[3]{x^2} + x\sqrt{x}$.

Soln. From $f(x) = x^{2/3} + x^{3/2}$, we see

$$F(x) = \frac{3}{5}x^{5/3} + \frac{2}{5}x^{5/2} + C.$$

#14 p355 Find the most general antiderivative of

$$f(x) = \frac{3t^4 - t^3 + 6t^2}{t^4}.$$

Soln. First rewrite $f(t)$ as

$$f(t) = 3 - \frac{1}{t} + 6t^{-2}.$$

From this, we see

$$F(t) = 3t - \ln|t| - \frac{6}{t} + C$$

#22p p356 Find a general antiderivative of

$$f(x) = \frac{2x^2+5}{x^2+1}.$$

Soln.

$$\begin{array}{r|l} & 2 \\ \hline x^2+1 & 2x^2 + 0x + 5 \\ & 2x^2 + 2 \\ \hline & 3 \end{array} \Rightarrow f(x) = 2 + \frac{3}{x^2+1}$$

Or, $f(x) = \frac{2x^2+5}{x^2+1} = \frac{2x^2+2-2+5}{x^2+1} = \frac{2(x^2+1)+3}{x^2+1} = 2 + \frac{3}{x^2+1}$.

So,

$$F(x) = 2x + 3\tan^{-1}x + C$$

#62 p357 Find the position of a particle given that

$a(t) = 3\cos t - 2\sin t$, $s(0) = 0$, $v(0) = 4$.

Solution. Since $\frac{d}{dt}v(t) = a(t)$, we have

$$\frac{dv}{dt} = 3\cos t - 2\sin t.$$

So, $v(t) = 3\sin t + 2\cos t + k.$

Since $\frac{d}{dt}s(t) = v(t)$, $\frac{ds}{dt} = 3\sin t + 2\cos t + k.$

Hence, $s(t) = -3\cos t + 2\sin t + kt + c$

As $s(0) = 0$, $-3\cos 0 + 2\sin 0 + k(0) + c = 0 \Rightarrow \underline{c = 3.}$

And so $s(t) = -3\cos t + 2\sin t + kt + 3.$

Thus, $v(t) = 3\sin t + 2\cos t + k.$

As $v(0) = 4$, $3\sin 0 + 2\cos 0 + k = 4 \Rightarrow \underline{k = 4-2 = 2}$

Therefore, $\boxed{s(t) = 2\sin t - 3\cos t + 2t + 3}$

#74 p357 A car is traveling at 50 mph when the brakes are fully applied, producing a constant deceleration of 22 ft/s². What is the distance traveled before the car comes to a stop?

Soln. Let $s(t)$ be the distance traveled (in feet) t seconds after the brakes have been applied.

Given. $\frac{d}{dt}v(t) = -22\,\frac{ft}{s^2}$ where $v(0) = 50\,\frac{miles}{hr}$ and $s(0) = 0$ miles

Objective: Find s when $v=0$.

Soln. $v(0) = 50\,\frac{miles}{hr} = \frac{50\,miles}{hr} \cdot \frac{hr}{60\,min} \cdot \frac{min}{60\,s} \cdot \frac{5280\,ft}{miles}$

$$= \frac{50(5280)}{3600}\,\frac{ft}{s} = \frac{5(528)}{36}\,\frac{ft}{s}\ \ \frac{5(44)}{3}\,\frac{ft}{s} = \frac{220}{3}\,\frac{ft}{s}.$$

Since

$$\frac{dv}{dt} = -22,$$

$v(t) = -22t + C$. Since $v(0) = \frac{220}{3}$, $C = \frac{220}{3}$

Thus $v(t) = -22t + \frac{220}{3}$.

The car stops when $v(0) = 0$: So

$-22t + \frac{220}{3} = 0 \implies t = \frac{220}{22(3)} = \frac{10}{3}\,s$.

As $\frac{ds}{dt} = -22t + \frac{220}{3}$,

$s(t) = -11t^2 + \frac{220}{3}t + K$. Since $s(0) = 0$,

$s(t) = -11t^2 + \frac{220}{3}t$.

Thus, $s\left(\frac{10}{3}\right) = -11\left(\frac{10}{3}\right)^2 + \frac{220}{3}\left(\frac{10}{3}\right) = \frac{-1100 + 2200}{9} = \frac{1100}{9}$

So, car stops in $\frac{1100}{9}$ ft $= 122.2$ ft

Braking/Stopping Distances

MPH	Ft./Sec.	Braking Deceleration Distance	Perception Reaction Distance	Total Stopping Distance
10	14.7	5	22	27
15	22	11	33	44
20	29.3	19	44	63
25	36	30	55	85
30	44	43	66	109
35	51.3	59	77	136
40	58.7	76	88	164
45	66	97	99	196
50	73.3	119	110	229
55	80.7	144	121	265
60	88	172	132	304
65	95.3	202	143	345
70	102.7	234	154	388
75	110	268	165	433
80	117.3	305	176	481
85	124.7	345	187	532
90	132	386	198	584