>

Areas and Riemann Sums

> *restart*

> *with*(*Student*[*Calculus1*]) :

>

Problem. Estimate the area of region enclosed by $y = x^2 + 1$, the x-axis, and the lines $x = 0$, and $x = 1$.

> $f := x \to 1 + x^2$

$$f := x \to 1 + x^2 \quad (1)$$

> $f(x)$

$$x^2 + 1 \quad (2)$$

> $a := 0;\ b := 1$

$$a := 0$$
$$b := 1 \quad (3)$$

>

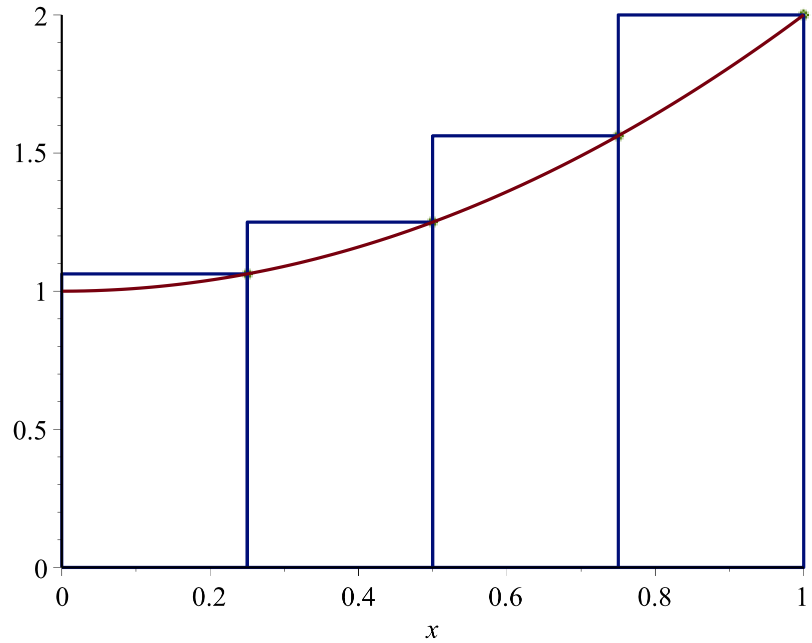
Let us partition the interval $[0, 1]$ into 4 subintervals of equal width:

> $N := 4$

$$N := 4 \quad (4)$$

>

> *RiemannSum*($f(x)$, x = a ..b, output = plot, partition = N, method = right)



A right Riemann sum approximation of $\int_0^1 f(x)\,dx$, where $f(x) = x^2 + 1$ and the partition is uniform. The approximate value of the integral is 1.468750000. Number of subintervals used: 4.

\>

\> *RiemannSum*($f(x)$, x = a ..b, output = sum, partition = N, method = right)

$$\frac{1}{4} \sum_{i=1}^{4} \left(\frac{1}{16} i^2 + 1 \right) \quad \textbf{(5)}$$

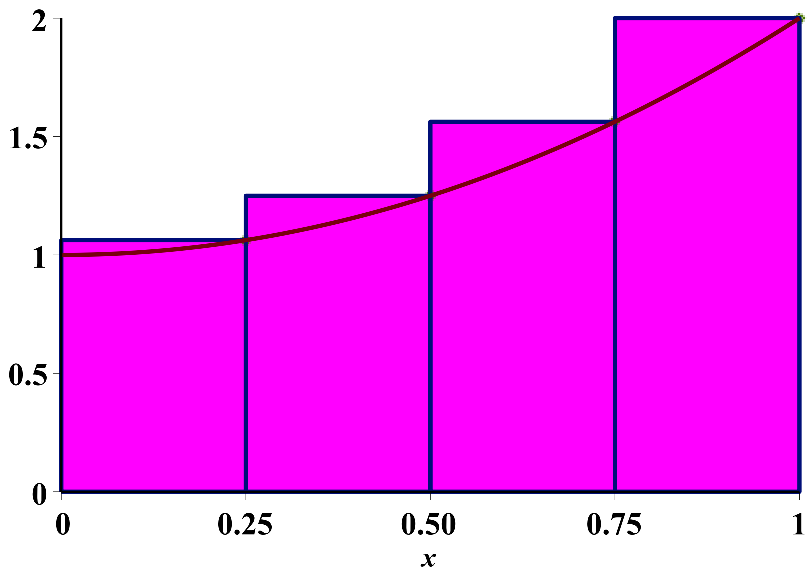
\>

\> *RiemannSum*($f(x)$, x = a ..b, output = value, partition = N, method = right)

$$\frac{47}{32} \quad \textbf{(6)}$$

\>

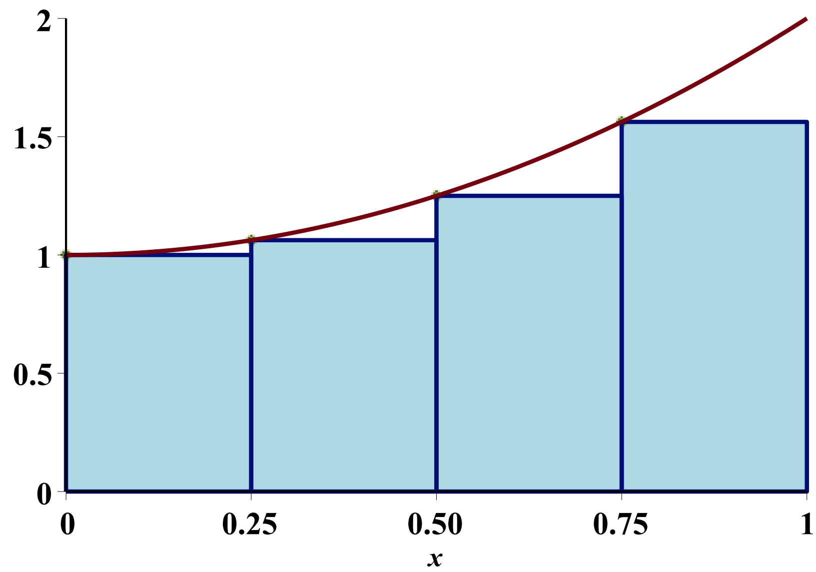
\> *RiemannSum*($f(x)$, $x = a..b$, *output* = *plot*, *partition* = *N*, *method* = *right*, *boxoptions* = [*filled* = [*color* = *magenta*, *transparency* = .9]], tickmarks = [5, 5], *thickness* = 2, *font* = [*Roman*, *bold*, 12], *labelfont* = [*Roman*, 10])



A right Riemann sum approximation of $\int_0^1 f(x)\,dx$, where $f(x) = x^2 + 1$ and the partition is uniform. The approximate value of the integral is 1.468750000. Number of subintervals used: 4.

>

> *RiemannSum*(*f*(*x*), *x* = *a*..*b*, *output* = *plot*, *partition* = *N*, *method* = *left*, *boxoptions* = [*filled* = [*color* = "LightBlue", *transparency* = .4]], tickmarks = [5, 5], *thickness* = 2, *font* = [*Roman*, *bold*, 12], *labelfont* = [*Roman*, 10])



A left Riemann sum approximation of $\int_0^1 f(x)\,dx$, where $f(x) = x^2 + 1$ and the partition is uniform. The approximate value of the integral is 1.218750000. Number of subintervals used: 4.

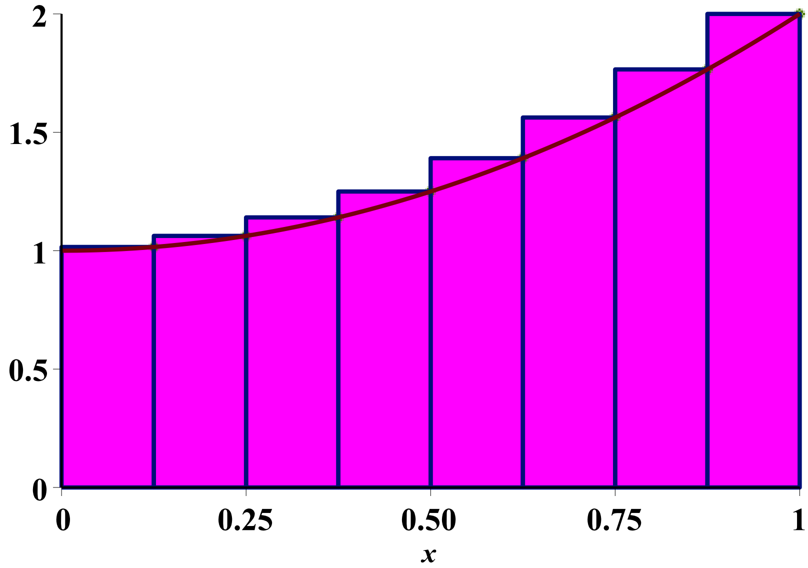
>

Now estimate the area using a partition of 8 subintervals of equal width:

> $N := 8$

$$N := 8 \qquad (7)$$

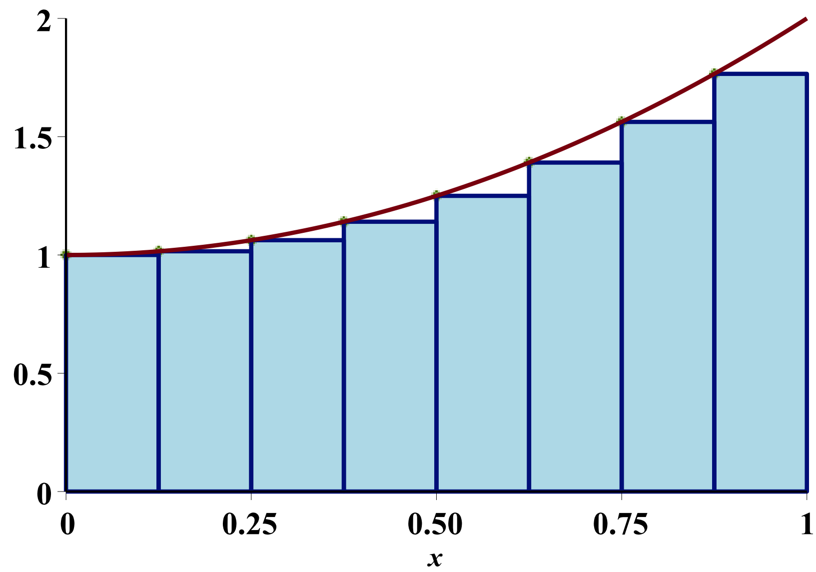
> $RiemannSum(f(x), x = a..b, output = plot, partition = N, method = right, boxoptions = [filled = [color = magenta, transparency = .9]], \text{tickmarks} = [5, 5], thickness = 2, font = [Roman, bold, 12], labelfont = [Roman, 10])$



A right Riemann sum approximation of $\int_0^1 f(x)\,dx$, where $f(x) = x^2 + 1$ and the partition is uniform. The approximate value of the integral is 1.398437500. Number of subintervals used: 8.

>

> $RiemannSum(f(x), x = a..b, output = plot, partition = N, method = left, boxoptions = [filled = [color = \text{"LightBlue"}, transparency = .4]], \text{tickmarks} = [5, 5], thickness = 2, font = [Roman, bold, 12], labelfont = [Roman, 10])$



A left Riemann sum approximation of $\int_0^1 f(x)\,dx$, where $f(x) = x^2 + 1$ and the partition is uniform. The approximate value of the integral is 1.273437500. Number of subintervals used: 8.

>

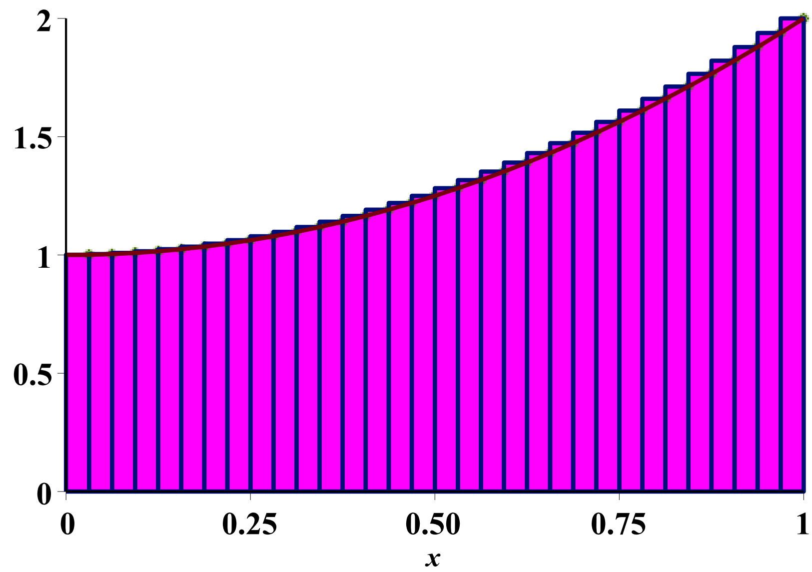
Now estimate the area using a partition of 32 subintervals of equal width:

> $N := 32$

$$N := 32 \quad (8)$$

>

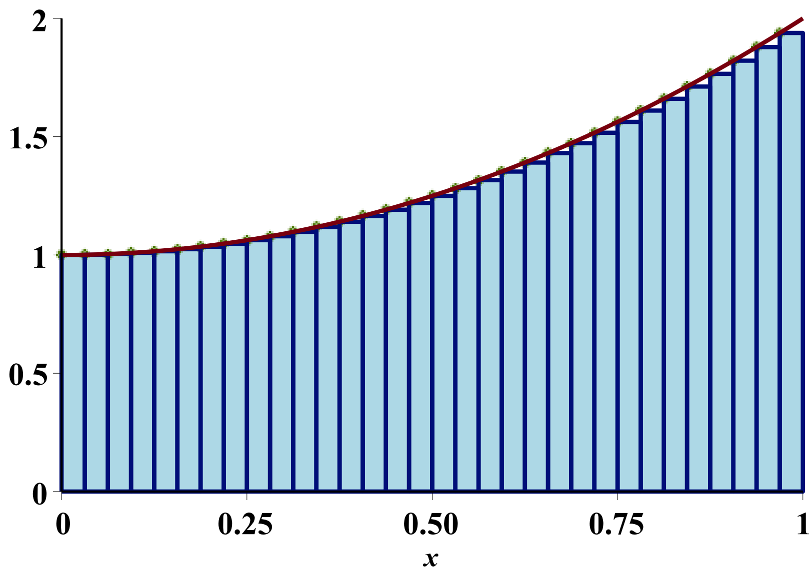
> $RiemannSum(f(x), x = a..b, output = plot, partition = N, method = right, boxoptions = [filled = [color = magenta, transparency = .9]], \text{tickmarks} = [5, 5], thickness = 2, font = [Roman, bold, 12], labelfont = [Roman, 10])$



A right Riemann sum approximation of $\int_0^1 f(x)\,dx$, where $f(x) = x^2 + 1$ and the partition is uniform. The approximate value of the integral is 1.349121093. Number of subintervals used: 32.

\>

\> $RiemannSum(f(x), x = a..b, output = plot, partition = N, method = left, boxoptions = [filled = [color = \text{"LightBlue"}, transparency = .4]], \text{tickmarks} = [5, 5], thickness = 2, font = [Roman, bold, 12], labelfont = [Roman, 10])$



A left Riemann sum approximation of $\int_0^1 f(x)\,dx$, where $f(x) = x^2 + 1$ and the partition is uniform. The approximate value of the integral is 1.317871093. Number of subintervals used: 32.

>

*** END ***

>