Math 112 - Quiz 15
November 12, 2018

Name Key

The following problems were selected from the practice problems, but some were slightly altered to simplify the algebra. Label answers appropriately. Show your work.

1. Differentiate $f(x) = (x-1)e^x$ and simplify. (2 pts.)

$f'(x) = (x-1)\frac{d}{dx}e^x + e^x\frac{d}{dx}(x-1)$ (Product rule)

$= (x-1)e^x + e^x(1) = xe^x - e^x + e^x = xe^x.$

$\boxed{f'(x) = xe^x}$

2. Differentiate $y = \dfrac{x^2}{x-1}$ and simplify. (2 pts.)

$y' = \frac{d}{dx}\left(\frac{x^2}{x-1}\right) = \frac{(x-1)\frac{d}{dx}x^2 - x^2\frac{d}{dx}(x-1)}{(x-1)^2}$ (Quotient Rule)

$= \frac{(x-1)(2x) - x^2(1)}{(x-1)^2} = \frac{2x^2 - 2x - x^2}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2}.$

$y' = \frac{x^2 - 2x}{(x-1)^2}$ or $y' = \frac{x(x-2)}{(x-1)^2}$

3. Find the points on the curve $y = 2x^3 + 3x^2 - 12x + 1$ where the tangent line is horizontal. (2 pts.)

A tangent line to a curve is horizontal where $y' = 0$.

Since $y' = \frac{d}{dx}(2x^3 + 3x^2 - 12x + 1) = 6x^2 + 6x - 12$, solve

$6x^2 + 6x - 12 = 0.$

Factoring, $6(x^2 + x - 2) = 0 \Rightarrow 6(x+2)(x-1) = 0$

Thus $y' = 0$ when $x = -2$ and $x = 1$.

When $x = 1$, $y = 2(1)^3 + 3(1)^2 - 12(1) + 1 = 2 + 3 - 12 + 1 = -6.$

When $x = -2$, $y = 2(-2)^3 + 3(-2)^2 - 12(-2) + 1 = -16 + 12 + 24 + 1 = 21.$

So, the points are $(1,-6)$ and $(-2,21)$.

4. Let $f(5) = 1$, $f'(5) = 4$, $g(5) = -3$, and $g'(5) = 2$. Find the value of $(g/f)'(5)$; that is, of (2 pts.)

$$\frac{d}{dx}\left(\frac{g(x)}{f(x)}\right)\Bigg|_{x=5}$$

By the Quotient Rule,

$$\frac{d}{dx}\left(\frac{g(x)}{f(x)}\right) = \frac{f(x)\,g'(x) - g(x)\,f'(x)}{[f(x)]^2}.$$

Hence,

$$\frac{d}{dx}\left(\frac{g(x)}{f(x)}\right)\Bigg|_{x=5} = \frac{f(5)\,g'(5) - g(5)\,f'(5)}{[f(5)]^2}$$

$$= \frac{(1)(2) - (-3)(4)}{1^2}$$

$$= \boxed{14}$$

5. Let g be a differentiable function. Find an expression for the derivative of $y = xg(x)$. (2 pts.)

By the Product Rule,

$$y' = x\frac{d}{dx}g(x) + g(x)\frac{d}{dx}x$$

$$= x\,g'(x) + g(x)\cdot 1$$

$$= x\,g'(x) + g(x). \quad \Rightarrow \quad y' = x\,g'(x) + g(x)$$

Extra credit. (1 pt.)

Complete the statement by filling in the blank. In the last class, we proved the following theorem: If a function f is differentiable at a point a, then f is continuous at $x=a$.

See class notes or Theorem 4 on p. 156

The answer "f is continuous" is not correct b/c the differentiability of f at a point does not imply that f is continuous everywhere.