Math 112 - Quiz 7
October 1, 2018

Name Key

No calculators. Show your work.

1. Find the exact value of $\log_{10} 40 + \log_{10} 2.5$. (2 pts.)

$$\log_{10} 40 + \log_{10} 2.5 = \log_{10}[(40)(2.5)] = \log_{10}(100) = \boxed{2}.$$

$(b/c\ 10^2 = 100)$

2. Fill in the blanks:

Let f be a $1-1$ function with domain A and range B. Then its inverse function f^{-1} has domain $\underline{B}$ and range $\underline{A}$ and is defined by

$$f^{-1}(y) = x \iff \underline{f(x)=y}$$

for any y in B. (2 pts.)

3. Fill in the blanks:

$\ln e = \underline{1}$. b/c $\ln e = \log_e e = 1$ since $e^1 = e$. (1 pt.)

Let x and y be positive numbers. Then $\log_3(xy) = \underline{\log_3 x + \log_3 y}$. (1 pt.)

4. Find a formula for the inverse of the function $y = \ln(x+3)$. (2 pts.)

Since $\ln(x+3) = \log_e(x+3)$,
$\log_e(x+3) = y$.

Thus, $e^y = x+3 \Rightarrow x = e^y - 3$.

Interchanging x & y, we have
$y = e^x - 3$. $\therefore f^{-1}(x) = e^x - 3$

Or: Exponentiate both sides:
$e^y = e^{\ln(x+3)}$.

Since $e^{\ln(x+3)} = x+3$,
$e^y = x+3$.

Solving for x: $x = e^y - 3$.

$\therefore f^{-1}(x) = e^x - 3$

5. Solve the following equation for x: $2^{x-5} = 3$. (2 pts.)

Take the logarithm with base 2 of both sides:

$$\log_2 2^{x-5} = \log_2 3.$$

So, $x - 5 = \log_2 3$.

Hence, $x = 5 + \log_2 3$

Or: Take the natural logarithm of both sides:

$$\ln 2^{x-5} = \ln 3.$$

So, $(x-5)\ln 2 = \ln 3$.

$\Rightarrow x - 5 = \dfrac{\ln 3}{\ln 2} \Rightarrow x = 5 + \dfrac{\ln 3}{\ln 2}$

The two answers are equal b/c

$$\log_2 3 = \frac{\ln 3}{\ln 2}.$$

See the "change of base formula".