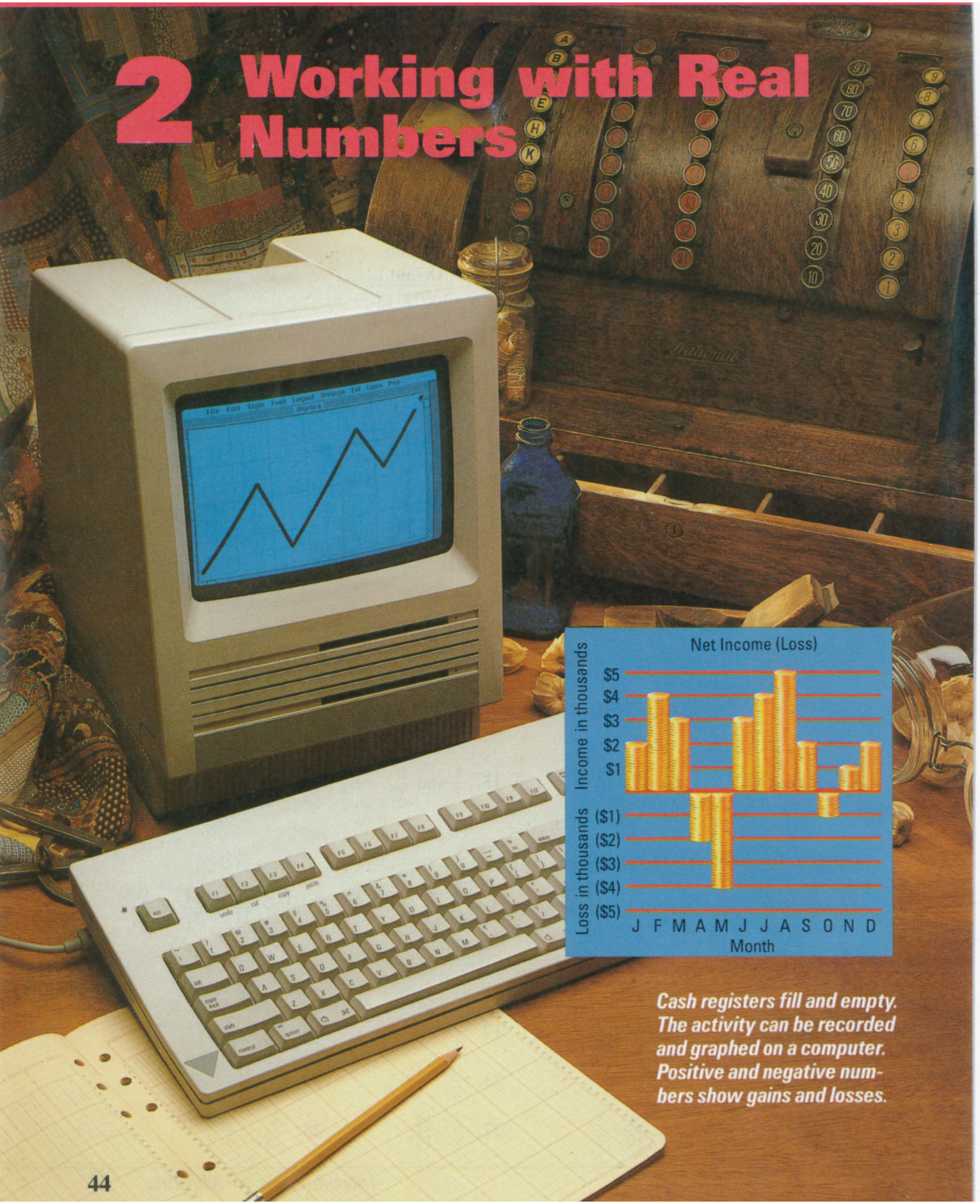
2 Working with Real Numbers



Cash registers fill and empty. The activity can be recorded and graphed on a computer. Positive and negative numbers show gains and losses.

Addition and Subtraction

2-1 Basic Assumptions

Objective To use number properties to simplify expressions.

This chapter develops rules for working with positive and negative numbers. The two principal operations are addition and multiplication. Subtraction is related to addition. Later in this chapter you will see that division is related to multiplication.

The rules for working with real numbers are based on several properties that you can accept as facts.

Closure Properties

For all real numbers a and b:

$a + b$ is a unique real number.

ab is a unique real number.

The sum and product of any two real numbers are also real numbers. Moreover, they are *unique*. (This means there is one and only one possible answer when you add or multiply two real numbers.)

Commutative Properties

For all real numbers a and b:

$a + b = b + a$ Example: $2 + 3 = 3 + 2$

$ab = ba$ Example: $4 \cdot 5 = 5 \cdot 4$

The order in which you add or multiply two numbers does not affect the result.

Associative Properties

For all real numbers a, b, and c:

$(a + b) + c = a + (b + c)$ Example: $(5 + 6) + 7 = 5 + (6 + 7)$

$(ab)c = a(bc)$ Example: $(2 \cdot 3)4 = 2(3 \cdot 4)$

When you add or multiply any three real numbers, the grouping (or association) of the numbers does not affect the result.

In the sum $a + b$, a and b are called **terms.** In the product ab, a and b are called **factors.** To find the sum or product of several numbers, you work with two numbers at a time. Examples 1 and 2 show how the commutative and associative properties can sometimes help you to simplify expressions.

Example 1 Simplify $43 + 78 + 7$.

Solution The rule for order of operations tells you to add from left to right: $(43 + 78) + 7$. For mental math it is easiest to add 43 and 7 first. The steps you do almost without thinking about them are shown below. Each step is justified by a property.

$(43 + 78) + 7 = (78 + 43) + 7$	Commutative property of addition
$= 78 + (43 + 7)$	Associative property of addition
$= 78 + 50$	Substitution of 50 for $43 + 7$
$= 128$	Substitution of 128 for $78 + 50$

Example 2 Simplify: **a.** $7 + 9m + 5$ **b.** $(2x)(3y)(4z)$

Solution **a.** $7 + 9m + 5 = 9m + (7 + 5)$
$= 9m + 12$ ***Answer***

b. $(2x)(3y)(4z) = (2 \cdot 3 \cdot 4)(xyz)$
$= 24xyz$ ***Answer***

To simplify expressions, you often use the properties listed below.

Properties of Equality

For all real numbers a, b, and c:

Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.

Throughout the rest of this book, the domain of each variable is the set of real numbers unless otherwise specified.

Oral Exercises

Simplify each expression. Use the commutative and associative properties to reorder and regroup.

1. $(5 \cdot 83) \cdot 2$ **2.** $(47 + 39) + 3$ **3.** $17 + 89 + 3 + 11$

4. $4 + 5x + 2$ **5.** $9 + 3y + 4$ **6.** $a + 1 + b + 2$

7. $2(7a)$ **8.** $(5n)4$ **9.** $(3x)(2y)$

10. $(5a)(4b)$ **11.** $(3p)(5q)(2r)$ **12.** $r(3s)(4t)$

Name the property illustrated.

13. $3(14) = (14)3$
14. $(74 + 99) + 1 = 74 + (99 + 1)$
15. $\frac{1}{6} + 9 = 9 + \frac{1}{6}$
16. $\frac{1}{5}(15w) = \left(\frac{1}{5} \cdot 15\right)w$
17. $2.1 + y = y + 2.1$
18. $3(8 \cdot 0) = (3 \cdot 8)0$
19. If $7m = 35$, then $35 = 7m$.
20. If $3 + p = 12$, then $12 = 3 + p$.
21. Every real number is equal to itself.
22. $(5 + q) + (-3) = (q + 5) + (-3)$
23. If $w + 2 = 7$ and $7 = 5 + 2$, then $w + 2 = 5 + 2$.
24. There is only one real number that is the sum of 0.4 and 2.6.
25. $(r + 37) + 23 + s = r + (37 + 23) + s$

Written Exercises

Simplify.

A

1. $275 + 52 + 25 + 8$
2. $803 + 26 + 47 + 24$
3. $2 \cdot 21 \cdot 5 \cdot 3$
4. $50 \cdot 3 \cdot 3 \cdot 20$
5. $25 \cdot 74 \cdot 2 \cdot 2$
6. $8 \cdot 17 \cdot 9 \cdot 25$
7. $6\frac{1}{2} + 4\frac{1}{3} + 1\frac{1}{2} + \frac{2}{3}$
8. $56\frac{7}{8} + \frac{3}{5} + \frac{1}{8} + 4\frac{2}{5}$
9. $0.1 + 1.8 + 5.9 + 0.2$
10. $4.75 + 2.95 + 1.05 + 10.25$
11. $3 + 7y + 4$
12. $8 + 9z + 4$
13. $5 + 2x + 1$
14. $5 + 3w + 2$
15. $2(5a)$
16. $3(4n)$
17. $(7y)(5z)$
18. $(6m)(4n)$
19. $(8x)(2y)(3z)$
20. $(3p)(4q)(6r)$

Sample $5x + 3 + 4y + 7 = 5x + 4y + (3 + 7)$
$= 5x + 4y + 10$ ***Answer***

21. $a + 3 + b + 4$
22. $7 + x + y + 4$
23. $4a + 5 + 3n + 1$
24. $9m + 2 + 7n + 3$

B

25. $5 + 7x + 3 + 4y + z$
26. $8p + 4 + 3q + 87 + 96$
27. $(25b)(25c)(4d)(4e)$
28. $(5a)(5b)(5c)(2d)$

Name the property illustrated.

29. $pq = qp$
30. $(25 + 84) + 16 = 25 + (84 + 16)$
31. $x + y = y + x$
32. If $3.2 = 8y$, then $8y = 3.2$.
33. $(3r)s = 3(rs)$
34. If $x = -3$ and $-3 = z$, then $x = z$.

Name the property that justifies each step.

35. $57 + (25 + 13) = 57 + (13 + 25)$ **a.** ___?___
$= (57 + 13) + 25$ **b.** ___?___
$= 70 + 25$ **c.** ___?___
$= 95$ **d.** ___?___

36. $25 \cdot (37 \cdot 8) = 25 \cdot (8 \cdot 37)$ **a.** ___?___
$= (25 \cdot 8) \cdot 37$ **b.** ___?___
$= 200 \cdot 37$ **c.** ___?___
$= (100 \cdot 2) \cdot 37$ **d.** ___?___
$= 100 \cdot (2 \cdot 37)$ **e.** ___?___
$= 100 \cdot 74$ **f.** ___?___
$= 7400$ **g.** ___?___

C **37. a.** Find the values of $(7 - 4) - 2$ and $7 - (4 - 2)$.
b. Is subtraction of real numbers associative?

38. a. Find the values of $(48 \div 8) \div 2$ and $48 \div (8 \div 2)$.
b. Is division of real numbers associative?

39. Is division of real numbers commutative? Give a convincing argument to justify your answer.

40. A set of numbers is said to be *closed* under an operation if the result of combining *any* two numbers in the set results in a number that is also in the set. Decide whether or not each set is closed under the operation.
a. {positive integers}; $\div$
b. {odd integers}; $\times$
c. {odd integers}; $+$
d. {integers ending in 4 or 6}; $\times$

Mixed Review Exercises

Evaluate if $a = 3$, $x = 5$, $y = 2$, and $z = 6$.

1. $\dfrac{2a - y}{4y - z}$
2. $5z(x - y)$
3. $\dfrac{3a + x}{4z - (y + 1)}$

Evaluate if $a = -4$, $b = 2$, and $x = -3$.

4. $|a| + |x| - b$
5. $2|a| + 7|x|$
6. $2|x| - |b| + |a|$
7. $10|x| - 3|b|$
8. $2|b| + 3|a|$
9. $|-x - 1| - |b - 1|$

Challenge

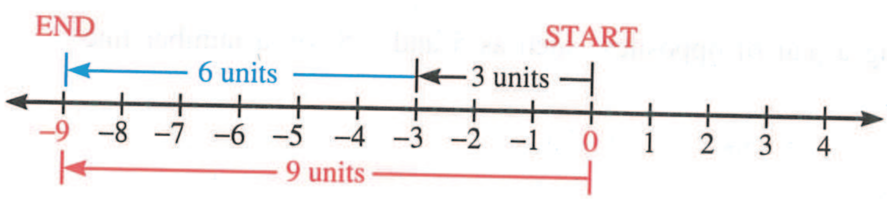
Change for a dollar consisted of 15 coins (pennies, nickels, and dimes only). At least one of each type of coin was used. How was this done?

2-2 Addition on a Number Line

Objective To add real numbers using a number line or properties about opposites.

You already know how to add two positive numbers. You can use a number line to help you find the sum of *any* two real numbers. Moves to the left represent negative numbers; moves to the right represent positive numbers.

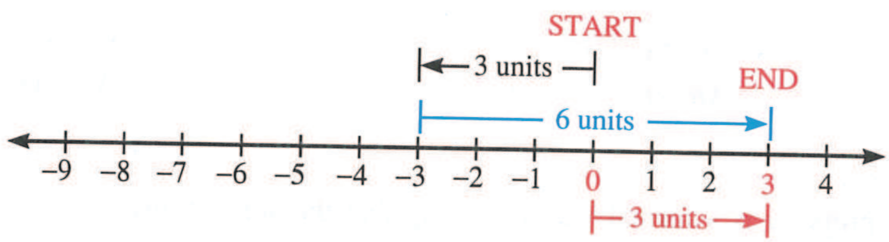
To find the sum of -3 and -6, first draw a number line. Then, starting at the origin, move your pencil 3 units to the left along your number line. From that position, move your pencil 6 more units to the left. Together, the two moves result in a move of 9 units to the left from the origin. The arrows in the diagram below show the moves.



This shows that $-3 + (-6) = -9$.

Note the use of parentheses in "$-3 + (-6)$" to separate the plus sign that means "add" from the minus sign that is part of the numeral for negative six.

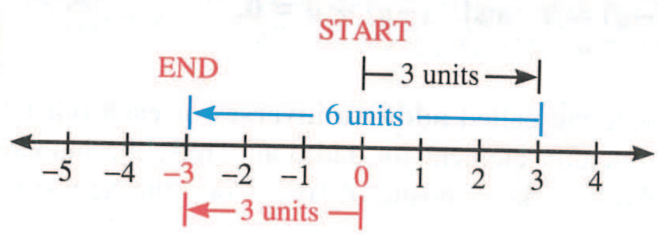
To find the sum $-3 + 6$, first move 3 units to the left from the origin. Then, from that position, move 6 units to the right. The two moves result in a move of 3 units to the right of the origin, as shown below.



This shows that $-3 + 6 = 3$.

Example 1 Simplify $3 + (-6)$.

Solution



$3 + (-6) = -3$ ***Answer***

Can you visualize $-3 + 0$ on a number line? Think of "adding 0" to mean "moving no units." Then you can see that

$$-3 + 0 = -3 \quad \text{and} \quad 0 + (-3) = -3.$$

These equations illustrate the special property of zero for addition of real numbers: When 0 is added to any real number, the sum is *identical* to that number. We call 0 the **identity element for addition.**

Identity Property of Addition

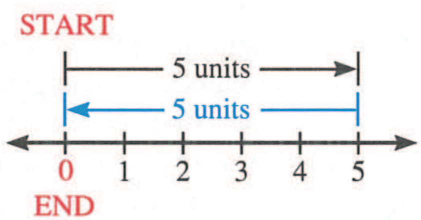
There is a unique real number 0 such that for every real number a,

$$\boldsymbol{a + 0 = a} \quad \text{and} \quad \boldsymbol{0 + a = a}.$$

Think of adding a pair of opposites, such as 5 and -5, on a number line as shown below.

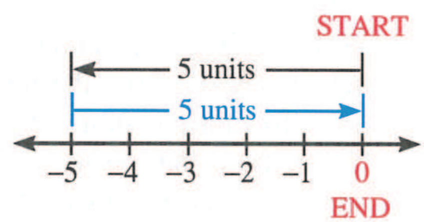
Example 2 Simplify: **a.** $5 + (-5)$ **b.** $-5 + 5$

Solution **a.**



$5 + (-5) = 0$
Answer

b.



$-5 + 5 = 0$
Answer

The following property is a formal way of saying that the sum of any number and its opposite is zero.

Property of Opposites

For every real number a, there is a unique real number $-a$ such that

$$\boldsymbol{a + (-a) = 0} \quad \text{and} \quad \boldsymbol{(-a) + a = 0}.$$

A number and its opposite are called **additive inverses** of each other because their sum is zero, the identity element for addition. Thus, the numeral -5 can be read "negative five," "the opposite of five," or "the additive inverse of five."

From the properties of addition, we can prove the following property of the opposite of a sum. (See Exercise 39.)

Property of the Opposite of a Sum

For all real numbers a and b:

$$-(a + b) = (-a) + (-b)$$

The opposite of a sum of real numbers is equal to the sum of the opposites of the numbers.

Example 3 Simplify: **a.** $-8 + (-3)$ **b.** $14 + (-5)$

Solution

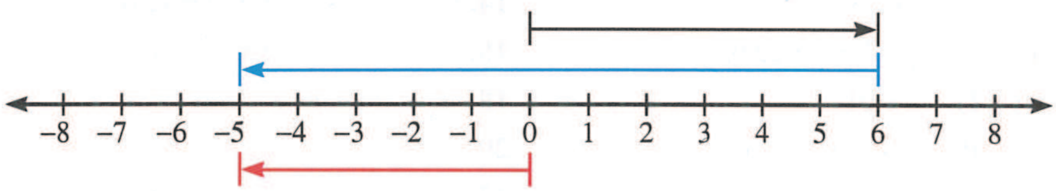
a. $-8 + (-3) = -(8 + 3)$
$= -11$ *Answer*

b. $14 + (-5) = (9 + 5) + (-5)$
$= 9 + [5 + (-5)]$
$= 9 + 0$
$= 9$ *Answer*

Oral Exercises

Give an addition statement illustrated by each diagram.

1.



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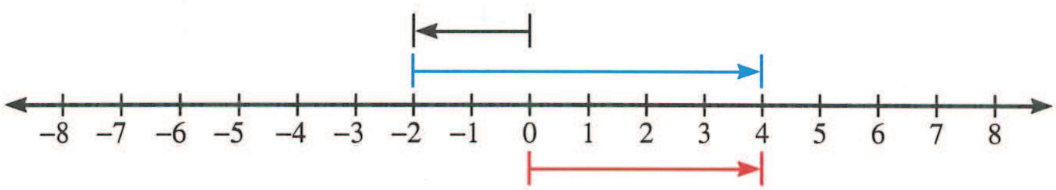

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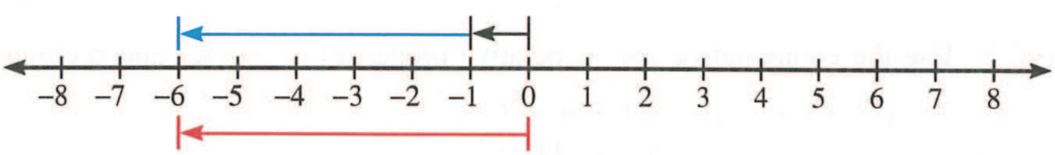

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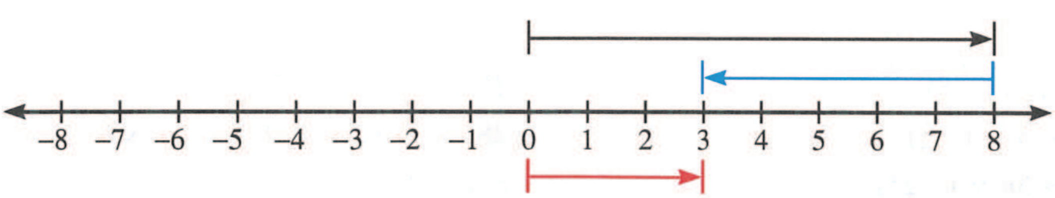


Simplify each expression. If necessary, think of moves along a number line.

5. $-2 + 0$ **6.** $0 + (-5)$ **7.** $-6 + 6$ **8.** $1 + (-1)$

9. $-4 + (-8)$ **10.** $-7 + (-1)$ **11.** $5 + (-1)$ **12.** $-4 + 9$

13. $-8 + 7$ **14.** $6 + (-4)$ **15.** $13 + (-18)$ **16.** $-1 + (-99)$

17. $102 + (-2)$ **18.** $-100 + 1$ **19.** $-10 + 0$ **20.** $-45 + 45$

Written Exercises

Simplify each expression. If necessary, draw a number line to help you.

A

1. $(-4 + 8) + 7$

2. $(-5 + 8) + 2$

3. $(-9 + 11) + (-2)$

4. $(-3 + 5) + (-2)$

5. $[3 + (-10)] + 6$

6. $[5 + (-15)] + 7$

7. $[-9 + (-8)] + 9$

8. $[-5 + (-23)] + 5$

9. $22 + [6 + (-11)]$

10. $32 + [8 + (-16)]$

11. $-26 + [-2 + (-8)]$

12. $-5 + [-12 + (-18)]$

13. $[37 + (-7)] + [1 + (-1)]$

14. $(-3 + 3) + [8 + (-12)]$

15. $[0 + (-8)] + [-7 + (-23)]$

16. $(-4 + 4) + [18 + (-8)]$

17. $-3 + (-4) + (-6)$

18. $-5 + (-7) + (-9)$

19. $-4 + (-10) + 9 + (-6)$

20. $-17 + 12 + (-4) + (-13)$

21. $-3.2 + (-4.6) + 5.4$

22. $5.9 + (-3.2) + (-7.8)$

23. $-\frac{3}{2} + 5 + \left(-\frac{7}{2}\right)$

24. $-\frac{9}{5} + (-3) + \left(-\frac{6}{5}\right)$

Simplify.

Sample $2 + (-3) + n + 7$

Solution Use the commutative and associative properties to reorder and regroup.

$$\begin{aligned} 2 + (-3) + n + 7 &= -1 + n + 7 \\ &= n + (-1) + 7 \\ &= n + 6 \quad \textit{Answer} \end{aligned}$$

25. $3 + x + (-8)$

26. $y + (-2) + 7$

27. $2n + 5 + (-5)$

28. $8 + 3n + (-12)$

29. $13 + 5n + (-21)$

30. $-7 + (-z) + 12$

B **31.** $3a + (-7) + 4b + (-6)$

32. $-2 + 4n + 9 + (-7)$

33. $7x + (-3) + (-y) + (-7)$

34. $-8 + 7c + 11 + (-3)$

C **35.** $b + [a + (-b)]$

36. $-m + (k + m)$

37. $-m + [-h + (m + h)]$

38. $[a + (-b)] + [b + (-a)]$

39. **a.** Show that $(a + b) + [(-a) + (-b)] = 0$.

b. The opposite of $(a + b)$ is $-(a + b)$. What property tells you that $(a + b) + [-(a + b)] = 0$?

c. Explain how the equation in part (a) and the equation in part (b) can be used together to prove the property of the opposite of a sum.

Mixed Review Exercises

Simplify.

1. $7 + 21 \div 7$

2. $|-6| + |3|$

3. $6 - [-(-2)]$

4. $\frac{4 + (3 \cdot 2)}{2}$

5. $\left|\frac{1}{3}\right| + \left|-\frac{1}{3}\right|$

6. $\left|-\frac{2}{3}\right| - \left|\frac{1}{3}\right|$

7. $63 + 25 + 17 + 15$

8. $25 \cdot 5 \cdot 20$

9. $(4a)(2b)(3c)$

10. $5\frac{2}{3} + 3\frac{1}{4} + 2\frac{1}{3} + 6\frac{3}{4}$

11. $0.7 + 2.4 + 6.3$

12. $2 \cdot 17 \cdot 5$

Challenge

Recall (page 35) that the triangular numbers are 1, 3, 6, 10, 15, and so on. 1, 4, 9, 16, 25, . . . are called square numbers because they can be represented by dots arranged to form squares.

1 4 9 16 25 and so on

1. What are the next two square numbers after 25?

2. Verify that each square number from 4 to 100 is the sum of two consecutive triangular numbers.

3. Illustrate Exercise 2 by dividing the square array of dots shown above for 25 into two triangular arrays.

2-3 Rules for Addition

Objective To add real numbers using rules for addition.

Perhaps you have already discovered these rules for adding positive and negative numbers without using a number line.

Rules for Addition	Examples
1. If a and b are both positive, then $a + b = \|a\| + \|b\|$.	$3 + 7 = 10$
2. If a and b are both negative, then $a + b = -(\|a\| + \|b\|)$.	$-6 + (-2) = -(6 + 2) = -8$
3. If a is positive and b is negative and a has the greater absolute value, then $a + b = \|a\| - \|b\|$.	$8 + (-5) = 8 - 5 = 3$
4. If a is positive and b is negative and b has the greater absolute value, then $a + b = -(\|b\| - \|a\|)$.	$4 + (-9) = -(9 - 4) = -5$
5. If a and b are opposites, then $a + b = 0$.	$2 + (-2) = 0$

A positive and a negative number are said to have *opposite* signs. The rules for adding two nonzero numbers can be restated as follows:

> If the numbers have the *same* sign, add their absolute values and prefix their common sign.
>
> If the numbers have *opposite* signs, subtract the lesser absolute value from the greater and prefix the sign of the number having the greater absolute value.

Example 1 shows two methods for adding several numbers.

Example 1 Simplify $7 + (-9) + 15 + (-12)$.

Solution 1 Add the numbers in order from left to right.

$$\underbrace{7 + (-9)} + 15 + (-12)$$
$$\underbrace{-2 \quad + 15} + (-12)$$
$$\underbrace{13 \quad + (-12)}$$
$$1 \qquad \textit{Answer}$$

Solution 2

1. Add positive numbers.	2. Add negative numbers.	3. Add the sums.
7	−9	22
15	−12	−21
22	−21	1
		Answer

Example 2 Add.

$$\begin{array}{r} -291 \\ 379 \\ 185 \\ \underline{-462} \end{array}$$

Solution

1. Add positive numbers.	2. Add negative numbers.	3. Add the sums.
379	−291	−753
185	−462	564
564	−753	−189
		Answer

Oral Exercises

Add.

1. $\begin{array}{r} 7 \\ \underline{7} \end{array}$ **2.** $\begin{array}{r} -4 \\ \underline{-3} \end{array}$ **3.** $\begin{array}{r} -9 \\ \underline{6} \end{array}$ **4.** $\begin{array}{r} 13 \\ \underline{-8} \end{array}$ **5.** $\begin{array}{r} -12 \\ \underline{-27} \end{array}$ **6.** $\begin{array}{r} -1 \\ \underline{10} \end{array}$

7. $\begin{array}{r} -16 \\ \underline{7} \end{array}$ **8.** $\begin{array}{r} -4 \\ \underline{-19} \end{array}$ **9.** $\begin{array}{r} -13 \\ \underline{53} \end{array}$ **10.** $\begin{array}{r} 35 \\ \underline{-75} \end{array}$ **11.** $\begin{array}{r} -98 \\ \underline{36} \end{array}$ **12.** $\begin{array}{r} 65 \\ \underline{-37} \end{array}$

Simplify.

13. $-8 + (-13)$ **14.** $-18 + 8$ **15.** $14 + (-15)$

16. $15 + (-9)$ **17.** $3 + (-12)$ **18.** $-5 + 22$

19. $4 + (-1) + (-3)$ **20.** $-4 + (-9) + 9$ **21.** $-3 + (-6) + 9$

Written Exercises

Add.

A **1.** $\begin{array}{r} -7 \\ \underline{6} \end{array}$ **2.** $\begin{array}{r} -5 \\ \underline{-6} \end{array}$ **3.** $\begin{array}{r} -34 \\ \underline{75} \end{array}$ **4.** $\begin{array}{r} 58 \\ \underline{-72} \end{array}$ **5.** $\begin{array}{r} 148 \\ -72 \\ -73 \\ \underline{-31} \end{array}$ **6.** $\begin{array}{r} -173 \\ 412 \\ -58 \\ \underline{-93} \end{array}$

Simplify.

7. $-12 + 7 + (-14) + 29$
8. $-16 + (-9) + 8 + 25$
9. $109 + (-56) + (-91) + 26$
10. $-206 + (-75) + 153 + 37$
11. $-[24 + (-5)] + [-(-2 + 6)]$
12. $[-7 + (-1)] + [-(-7 + 1)]$
13. $3.7 + 4.2 + (-2.3) + 0 + 6.4 + 12.8$
14. $-7.2 + 11.4 + (-8.1) + (-9.7) + 0.6$
15. $27 + 43 + (-14) + (-57) + 5 + (-36) + (-14)$
16. $46 + (-33) + 18 + 0 + (-93) + (-2) + (-34)$
17. $-\frac{1}{3} + \left(-1\frac{2}{3}\right) + 2$
18. $3 + \left(-\frac{5}{2}\right) + \left(-\frac{7}{2}\right)$
19. $-1\frac{3}{4} + 2\frac{1}{4}$
20. $-3\frac{2}{5} + \left(-1\frac{4}{5}\right)$
21. $-\frac{7}{8} + \left(-\frac{11}{8}\right)$
22. $\frac{16}{3} + \left(-\frac{10}{3}\right)$

Sample $5 + (-7) + (-x) + (-10) = -x + 5 + (-7) + (-10) = -x + (-12)$

23. $-3 + x + (-5) + 4$
24. $2 + (-9) + (-y) + (-11)$
25. $-4 + a + 4 + (-a)$
26. $21 + 4b + (-17) + (-12)$
27. $-[8 + (-5)] + (-c) + 3$
28. $-(-9) + 2y + (-9) + 6$
29. $x + 5 + [(-x) + (-14)]$
30. $4x + [8 + (-5) + (-7)]$

Evaluate each expression if $x = -3$, $y = 6$, and $z = -4$.

B

31. $x + y + (-1)$
32. $-21 + x + z$
33. $-15 + (-x) + y$
34. $-z + (-8) + y$
35. $1 + (-y) + z$
36. $-y + (-11) + x$
37. $|x + y + z|$
38. $|x + (-y) + z|$
39. $-|x + z + (-8)|$

Simplify.

C

40. $-b + [-a + (a + b)]$
41. $-(-x + y) + y$
42. $r + s + [-(r + s + t)]$
43. $a + [-(a + b)]$

Problems

a. Write a positive number or a negative number to represent each situation.
b. Compute the sum of the numbers.
c. Answer each question.

Sample A helicopter flying at an altitude of 2860 ft descended 120 ft and then rose 350 ft. What was its new altitude?

Solution **a.** 2860, −120, 350 **b.** $2860 + (-120) + 350 = 3090$ **c.** 3090 ft

A 1. An elevator started at the eighteenth floor. It then went down seven floors and up nine floors. At what floor was the elevator then located?

2. A submarine descended to a level 230 m below the surface of the ocean. Later it ascended 95 m and then dove 120 m. What was the new depth of the submarine?

3. Moira has $784 in her checking account. She deposits $96 and then writes two checks, one for $18 and the other for $44. What is the new balance in the account?

4. On five plays a football team lost 3 yards, gained 15, gained 8, lost 9, and lost 4. What was the net yardage on the plays?

5. In September, the Drama Club treasury contained $698. The club then presented a fall play, for which they paid a royalty of $250. Scenery and costumes cost $1684, and programs and other expenses totaled $1316. The sale of tickets brought in $3700. How much money was in the treasury after the play was presented?

6. The stock of the Computer Research Corporation opened on Monday at $32 per share. It lost $5 that day, dropped another $3 on Tuesday, but then gained $4 on Wednesday and $2 on Thursday. On Friday it was unchanged. What was its closing price for the week?

7. During a four-day period at the Hotel Gran Via, the numbers of guests checking in and out were as follows: 32 in and 27 out, 28 in and 31 out, 12 in and 18 out, and 16 in and 25 out. How did the number of guests in the hotel at the end of the fourth day compare with the number at the start of the four-day period?

8. During their first year after opening a restaurant, the Habibs had a loss of $14,250. In their second year of operation, they broke even. During their third and fourth years, they had gains of $18,180 and $29,470, respectively. What was the restaurant's net gain or loss over the four-year period?

9. A delivery truck traveled 5 blocks west, 3 blocks north, 8 blocks east, and 12 blocks south. After this, where was the truck relative to its starting point?

10. During an unusual storm, the temperature fell 8° C, rose 5° C, fell 4° C, and then rose 6° C. If the temperature was 32° C at the outset of the storm, what was it after the storm was over?

B 11. The lowest point in Death Valley is 86 m below sea level. A helicopter flying at an altitude of 41 m above this point climbed 28 m and then dropped 37 m. At what altitude relative to sea level was it then flying?

12. The rim of a canyon is 156 ft below sea level. If a stranded hiker 71 ft below the rim fired a warning flare that rose 29 ft, to what altitude relative to sea level would the flare rise?

13. Leaving Westwood at 1:00 P.M., Coretta flew to Bay View. The flight took 1.5 hours, but the time zone for Bay View is 2 hours earlier than Westwood's zone. What time was it in Bay View when Coretta landed?

14. A passenger on a train traveling at 135 km/h walks toward the back of the train at a rate of 7 km/h. What is the passenger's rate of travel with respect to the ground?

Solve.

C **15.** Wes charged $175 for clothing and $287 for camera equipment. He then made two payments of $50 each to his charge account. The next bill showed that he owed a balance of $495.25, including $11.50 in interest charges. How much did Wes owe on the account before making his purchase?

16. The volume of water in a tank during a five-day period changed as follows: up 375 L, down 240 L, up 93 L, down 164 L, and down 157 L. What was the volume of water in the tank at the beginning of the five-day period if the final volume was 54 L?

Mixed Review Exercises

Simplify.

1. $5 - 9 \div 3$

2. $3 \cdot 5 \cdot 4 \cdot 20$

3. $(6 - 4 \div 2) \cdot 3 + 1$

4. $|-7| - 6$

5. $|-1.2| + 1.2$

6. $|-15| - |-6|$

7. $\frac{7 \cdot 6 + 3 \cdot 7}{(5 + 2)}$

8. $6\frac{1}{7} + 5\frac{2}{3} + 8\frac{6}{7}$

9. $3.2 + 1.0 + 4.8$

10. $[15 + (-5)] + 6$

11. $(-8 + 4) + (-6)$

12. $-3 + (-9) + 6 + (-5)$

Calculator Key-In

Most calculators have a change-sign key, +/−, that will change a number from positive to negative or from negative to positive. When adding two or more positive and negative numbers, you can add in the usual order, making a number negative by hitting the change-sign key after entering the number.

Exercises

Use your calculator to find the sum.

1. $198 + (-217)$

2. $-612 + 38$

3. $-418 + 27$

4. $-115.8 + 22.4$

5. $13.8 + (-21.9)$

6. $-319.2 + 45.5$

2-4 Subtracting Real Numbers

Objective To subtract real numbers and to simplify expressions involving differences.

The first column below lists a few examples of the subtraction of 2. The second column lists related examples of the addition of -2.

Subtracting 2	Adding -2
$3 - 2 = 1$	$3 + (-2) = 1$
$4 - 2 = 2$	$4 + (-2) = 2$
$5 - 2 = 3$	$5 + (-2) = 3$
$6 - 2 = 4$	$6 + (-2) = 4$

Comparing the entries in the two columns shows that subtracting 2 gives the same result as adding the opposite of 2. This suggests the following:

Definition of Subtraction

For all real numbers a and b, the **difference** $a - b$ is defined by

$$a - b = a + (-b).$$

To subtract b, add the opposite of b.

Example 1 Simplify.

a. $3 - 12$ **b.** $-7 - 1$ **c.** $-4 - (-10)$ **d.** $y - (y + 6)$

Solution

a. $3 - 12 = 3 + (-12) = -9$

b. $-7 - 1 = -7 + (-1) = -8$

c. $-4 - (-10) = -4 + 10 = 6$

d. $y - (y + 6) = y + [-(y + 6)] = y + (-y) + (-6) = -6$

Note that in part (d), the property of the opposite of a sum is used. The opposite of $(y + 6)$ is $-y + (-6)$.

Using the definition of subtraction, you may replace any difference with a sum. For example,

$$12 - 8 - 7 + 4 \quad \text{means} \quad 12 + (-8) + (-7) + 4.$$

As shown at the right, you may simplify this expression by grouping from left to right. This method is convenient if you are doing the work mentally or with a calculator.

$$\underbrace{12 + (-8)} + (-7) + 4$$
$$\underbrace{4 \quad + (-7)} + 4$$
$$\underbrace{-3 \quad + 4}$$
$$1$$

For written work, you may want to group positive terms and negative terms:

$$\begin{aligned} 12 - 8 - 7 + 4 &= 12 + (-8) + (-7) + 4 \\ &= (12 + 4) + [(-8) + (-7)] \\ &= 16 + (-15) = 1 \end{aligned}$$

Certain sums are usually replaced by differences. For example,

$9 + (-2x)$ is usually simplified to $9 - 2x$.

Example 2 Simplify: **a.** -30 decreased by -19
b. The opposite of the difference between a and b

Solution **a.** First write the expression: $-30 - (-19)$
Then simplify: $-30 - (-19) = -30 + 19 = -11$ ***Answer***

b. First write the expression: $-(a - b)$. Then simplify:

$-(a - b) = -[a + (-b)]$ (Definition of subtraction)
$= (-a) + [-(-b)]$ (Property of the opposite of a sum)
$= -a + b$ ***Answer*** (Property of opposites)

The property of the opposite of a sum and the definition of subtraction produce this useful result: *When you find the opposite of a sum or a difference, you change the sign of each term of the sum or difference.*

$$-(a + b) = (-a) + (-b) = -a - b \qquad -(a - b) = (-a) - (-b) = -a + b$$

Now that you understand the reasoning behind each step, you can simplify some expressions mentally and write only the results, as shown in Example 3.

Example 3 Simplify: **a.** $-(x - 4)$ **b.** $-(-c + 5 - n)$ **c.** $x - (y - 2)$

Solution **a.** $-x + 4$ **b.** $c - 5 + n$ **c.** $x - y + 2$

Caution: Subtraction is *not* commutative. $8 - 3 = 5$ but $3 - 8 = -5$

Subtraction is *not* associative. $(6 - 4) - 3 = -1$ but $6 - (4 - 3) = 5$

Oral Exercises

Simplify.

1. $2 - 4$ **2.** $19 - 12$ **3.** $7 - 5$ **4.** $7 - 27$
5. $0 - 8$ **6.** $0 - (-6)$ **7.** $-16 - 0$ **8.** $-8 - 2$
9. $3 - (-1)$ **10.** $7 - (-3)$ **11.** $-9 - (-13)$ **12.** $-6 - (-4)$
13. $x - (x + 1)$ **14.** $n - (n + 4)$ **15.** $-(a - 3)$ **16.** $-(w - 5)$

Written Exercises

Simplify.

A

1. $25 - 213$
2. $154 - 281$
3. $39 - (-32)$
4. $47 - (-49)$
5. $-19 - (-3)$
6. $-25 - (-9)$
7. $-2.8 - 4.4$
8. $-5.1 - 6.7$
9. $174 - (-24)$
10. $-106 - (-95)$
11. $1.91 - (-1.03)$
12. $2.95 - (-2.55)$
13. -21 decreased by 6
14. -6 decreased by -32
15. 24 less than -1
16. 15 less than -3
17. $132 - (72 - 61)$
18. $275 - (80 - 65)$
19. $234 - (56 - 87)$
20. $193 - (30 - 75)$
21. $(22 - 33) - (55 - 66)$
22. $(42 - 50) - (73 - 60)$
23. $(3 - 8) - (-15 + 19)$
24. $(42 - 33) - (-7 + 12)$
25. $106 - 492 + 776$
26. $910 - 939 - 201$
27. $12 - (-9) - [5 - (-4)]$
28. $-15 - 6 - [-7 - (-10)]$
29. $4 - 5 + 8 - 17 + 31$
30. $18 - 14 + 15 + 7 - 26$
31. $-6 - 19 + 4 - 8 + 20$
32. $-11 - 43 + 1 - 9 + 30$
33. The difference between 81 and -6, decreased by -29
34. -54 decreased by the difference between -37 and 15
35. -46 decreased by the difference between -23 and -61
36. The difference between -59 and 12, decreased by 72
37. $-(x - 7)$
38. $-(4 - y)$
39. $x - (x + 5)$
40. $8 - (y + 8)$
41. $5 - (b - 7)$
42. $n - (4 + n)$

Simplify.

B

43. The sum of 8 and y subtracted from z
44. The opposite of the sum of -7 and y
45. Five less than the difference between 7 and z
46. The opposite of the difference between x and 4
47. $7 + y - (7 - y) - y$
48. $s - (-3) - [s + (-3)] - 3$
49. $h + 8 - (-9 + h)$
50. $-(10 - k) - (k - 12)$
51. $(\pi + 2)$ subtracted from $(\pi - 17)$
52. $(7 - 2\pi)$ subtracted from $(11 - 2\pi)$

Evaluate each expression if $a = -4$, $b = 5$, and $c = -1$.

53. $c - |a - b|$
54. $b - c - |a|$
55. $|c - a| - b$
56. $|b - c| - a$
57. $a - |c| - (|a| - b)$
58. $|c| - b - (|a| - c)$
59. $(a - |c|) - |a| - b$
60. $(|c| - b) - |a| - c$

In Exercise 61, name the definition or property that justifies each step.

C **61. a.** $b + (a - b) = b + [a + (-b)]$ **a.** ___?___
$= b + [(-b) + a]$ **b.** ___?___
$= [b + (-b)] + a$ **c.** ___?___
$= 0 + a$ **d.** ___?___
$= a$ **e.** ___?___

b. Use the result of part (a) to complete the following sentence: $a - b$ is the number to add to b to obtain ___?___.

62. Suppose you are told that $b - a$ *is the opposite of* $a - b$. Explain how adding $b - a$ to $a - b$ can convince you that this statement is true.

Problems

Express the answer to each question as the difference between two real numbers. Compute the difference. Answer the question and interpret the sign of the answer.

Sample If astronauts traveled from the sunny side of the moon to the dark side, they would find the temperature 220° C lower than on the sunny side. If the temperature on the sunny side was 90° C, what was it on the dark side?

Solution $90 - 220 = -130$
The temperature on the dark side of the moon is 130 degrees *below* 0° C.

A **1.** Mars is 141.5 millions of miles from the sun. Earth is 48.6 millions of miles closer to the sun than Mars is. How far is Earth from the sun?

2. The highest recorded weather temperature on Earth is 58.0° C. The difference between that record and the lowest recorded weather temperature is 146.3° C. What is the record low?

3. The Roman poet Virgil was born in 70 B.C. How old was he on his birthday in 42 B.C.?

4. Pythagoras, the Greek mathematician, was born in 582 B.C. and died on his birthday in 497 B.C. How old was he when he died?

5. Krypton boils at 153.4° below 0° C and melts at 157.2° below 0° C. What is the difference between its boiling and melting points?

6. Including the wind-chill factor, the temperature at Council Bluffs was eight degrees below zero Celsius at midnight and seventeen below zero at dawn. What was the change in temperature?

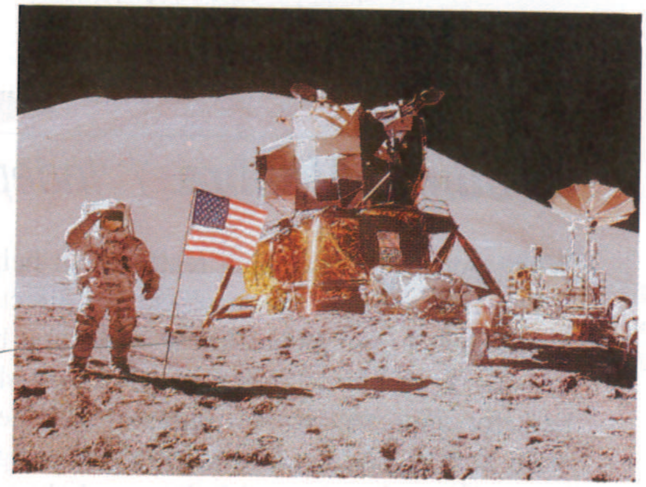
7. Becky Kasai took the subway from a stop 52 blocks east of Central Square to a stop 39 blocks west of Central Square. How many blocks did she ride?

8. Glynis drove a golf ball from a point 126 yd north of the ninth hole to a point 53 yd south of the hole. How far did the ball travel?

9. Find the difference in altitude between Mt. Rainier, Washington, 4392 m above sea level, and Death Valley, California, 86 m below sea level.

10. The difference in altitude between the highest and lowest points in Louisiana is 164.592 m. If Driskill Mountain, the highest point, is 163.068 m above sea level, what is the altitude of New Orleans, the lowest point?

B 11. In 1972, the Apollo space mission land rover weighed about 475 lb on Earth. Because of low gravity, it weighed 396 lb less on the moon. How much did the land rover weigh on the moon?



12. Alonzo has $22; Brad has $29; Caryl has $2; and Diane has $13. Alonzo owes Brad $15 and Caryl $3. Brad owes Caryl $18 and Diane $7. Diane owes Caryl $11 and Alonzo $5. When all debts are paid, how much money will each have?

Mixed Review Exercises

Simplify.

1. $|-9| + |7|$
2. $18 \cdot 3 \cdot 5 \cdot 2$
3. $4 + 7x + 3y + 5$
4. $\left|-\frac{2}{5}\right| - \left|\frac{1}{5}\right|$
5. $-\frac{5}{4} + \left(-\frac{7}{4}\right)$
6. $1\frac{1}{3} + \left(-3\frac{2}{3}\right)$
7. $[6 + (-12)] + 9$
8. $4.2 - 0.5 + (-3.2)$
9. $-6 + [-8 + (-4)]$
10. $-3.2 + 74 + (-2.8)$
11. $-41 + (-28) + 32 + 49$
12. $3 + (-7) + (-12) + (-x)$

Self-Test 1

Vocabulary

closure properties (p. 45)
commutative properties (p. 45)
associative properties (p. 45)
terms (p. 46)
factors (p. 46)
reflexive property (p. 46)
symmetric property (p. 46)
transitive property (p. 46)
identity element for addition (p. 50)
identity property of addition (p. 50)
property of opposites (p. 50)
additive inverses (p. 50)
property of the opposite of a sum (p. 51)
difference (p. 59)
subtraction (p. 59)

(Self-Test continues on next page.)

Simplify.

1. $9 \cdot 4 \cdot 7 \cdot 25$		**2.** $82 + 31 + x + 18$	**Obj. 2-1, p. 45**
3. $3 + (-4)$	**4.** $(-5 + 6) + 4$	**5.** $4 + 2x + (-5)$	**Obj. 2-2, p. 49**
6. $-13 + (-7)$		**7.** $-201 + (-19) + 20 + 180$	**Obj. 2-3, p. 54**
8. $12 - 26$		**9.** $9 - (y - 9)$	**Obj. 2-4, p. 59**

Check your answers with those at the back of the book.

Reading Algebra / *Independent Study*

When you read an algebra textbook, it helps to know the goals of a lesson. The lesson title and the lesson objective (directly below the title) give you a good idea of what you should know when you have finished reading.

Have you noticed that it's useful to skim each lesson first? As you read, look for important words, phrases, and ideas that are in **heavy type,** in *italics,* or in boxes. You need to know the highlighted ideas in order to understand the lesson. The glossary and index can help you find more information about important ideas.

Working through each example in a lesson can help you do some of the Oral and Written Exercises on your own. Doing the exercises will let you know whether you understand the lesson objectives.

The Self-Tests, with answers at the back of the book, will help you review a group of lessons. The Chapter Reviews, Chapter Tests, Cumulative Reviews, Mixed Reviews, and Mixed Problem Solving Reviews will also give you a good idea of your progress. If you do not understand a concept, and re-reading doesn't seem to help, it is a good idea to make a note of the concept so that you can discuss it with your teacher.

Exercises

Skim through Lessons 4-2 and 4-3 (pages 146–154). Then answer the following questions.

1. What should you be able to do when you have finished reading the text of these lessons?
2. What new words or phrases are introduced?
3. What is a monomial? a polynomial? Find the definitions of these words in your book.
4. Suppose that you had forgotten the definition of the term *variable*. Where could you look it up? On what page of your textbook is this word first used?
5. Lesson 4-2 covers adding and subtracting polynomials. Where could you find information about addition and subtraction in general?

Multiplication

2-5 The Distributive Property

Objective To use the distributive property to simplify expressions.

The cost of a certain model of cross-country skis is \$90. A pair of ski poles costs \$12. When Rita bought her family 4 sets of skis and poles, the total cost could have been calculated in two ways:

(1) Total cost:
$4 \times$ (cost of one set of skis and poles)
$4 \times (90 + 12) = 4 \times 102 = \$408$

(2) Total cost:
($4 \times$ cost of skis) + ($4 \times$ cost of poles)
$(4 \cdot 90) + (4 \cdot 12) = 360 + 48 = \$408$

Either way you compute it, the total cost is the same:

$$4(90 + 12) = (4 \cdot 90) + (4 \cdot 12)$$

Note that 4 is *distributed* as a multiplier of each term of the sum 90 + 12. This example illustrates another important property that we use when working with real numbers.

Distributive Property (of multiplication with respect to addition)

For all real numbers a, b, and c:

$$a(b + c) = ab + ac \quad \text{and} \quad (b + c)a = ba + ca.$$

Example 1 shows how the distributive property can make mental math easier.

Example 1

a. $5 \cdot 83 = 5(80 + 3)$
$= (5 \cdot 80) + (5 \cdot 3)$
$= 400 + 15$
$= 415$ ***Answer***

b. $6(9.5) = 6(9 + 0.5)$
$= (6 \cdot 9) + (6 \cdot 0.5)$
$= 54 + 3$
$= 57$ ***Answer***

c. $8\left(2\frac{1}{4}\right) = 8\left(2 + \frac{1}{4}\right)$
$= (8 \cdot 2) + \left(8 \cdot \frac{1}{4}\right)$
$= 16 + 2 = 18$ ***Answer***

Example 2 a. $5(x + 2) = 5 \cdot x + 5 \cdot 2$
$= 5x + 10$ ***Answer***

b. $(6y + 7)4 = 6y \cdot 4 + 7 \cdot 4$
$= 24y + 28$ ***Answer***

Multiplication is distributive with respect to subtraction as well as addition. For example,

$$2(8 - 3) = 2 \cdot 8 - 2 \cdot 3 = 16 - 6 = 10 \quad \text{and} \quad 4(x - 6) = 4 \cdot x - 4 \cdot 6 = 4x - 24.$$

This principle is stated in general below and will be proved in the next lesson. (See Exercise 60, page 73.)

Distributive Property (of multiplication with respect to subtraction)

For all real numbers a, b, and c:

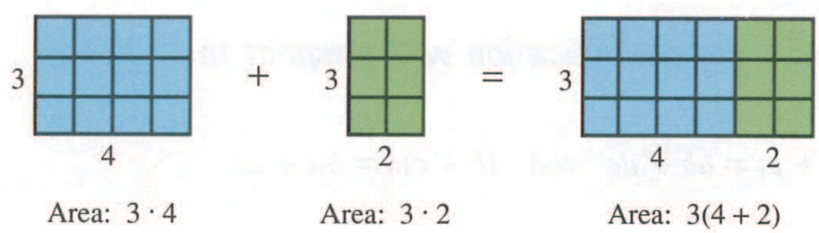
$$a(b - c) = ab - ac \quad \text{and} \quad (b - c)a = ba - ca.$$

By applying the symmetric property of equality, the distributive properties of multiplication can also be written in the following forms:

$$ab + ac = a(b + c) \qquad ba + ca = (b + c)a$$
$$ab - ac = a(b - c) \qquad ba - ca = (b - c)a$$

For example, the diagram below illustrates that

$$(3 \cdot 4) + (3 \cdot 2) = 3(4 + 2).$$



Example 3 Simplify $75 \cdot 17 + 25 \cdot 17$.

Solution $75 \cdot 17 + 25 \cdot 17 = (75 + 25)17$
$= (100)17$
$= 1700$ ***Answer***

Example 4 Show that $9x + 5x = 14x$ for every real number x.

Solution $9x + 5x = (9 + 5)x$ Distributive property
$= 14x$ Substitution principle

The properties of real numbers and equality guarantee that for all values of the variable x, $9x + 5x$ and $14x$ represent the same number. Therefore the two expressions are **equivalent.**

The expression $9x + 5x$ has two terms. The expression $14x$ has one term. Replacing an expression containing variables by an equivalent expression with as few terms as possible is called **simplifying the expression.**

Example 5 Simplify: **a.** $8y - 6y$ **b.** $3n + 7n - 4n$ **c.** $8(n - 1) + 3n$

Solution

a. $8y - 6y = (8 - 6)y$
$= 2y$ *Answer*

b. $3n + 7n - 4n = 10n - 4n$
$= 6n$ *Answer*

c. $8(n - 1) + 3n = 8 \cdot n - 8 \cdot 1 + 3n$
$= 8n - 8 + 3n$
$= 11n - 8$ *Answer*

Oral Exercises

Use the distributive property and mental math to simplify each expression.

1. $2(30 + 5)$ **2.** $7(40 + 1)$ **3.** $4(20 - 1)$ **4.** $5(60 - 2)$

5. $3 \cdot 99$ **6.** $8 \cdot 3\frac{1}{4}$ **7.** $10\left(3\frac{1}{5}\right)$ **8.** $5(7.2)$

For each expression give an equivalent expression without parentheses.

9. $5(a + 2)$ **10.** $3(x + 4)$ **11.** $6(n + 3)$ **12.** $4(a - 1)$

13. $5(2t - 3)$ **14.** $9(3n - 2)$ **15.** $8(b - c)$ **16.** $7(m + n)$

Simplify.

17. $6x + 16x$ **18.** $12y + 8y$ **19.** $2a + 5a$

20. $3b + b$ **21.** $(-2)x + (-6)x$ **22.** $(-3)y + (-8)y$

23. $11k - 15k$ **24.** $2c - 4c$ **25.** $3(1 + z) + 7$

Written Exercises

Simplify.

A **1.** $8(30 + 1)$ **2.** $7(60 + 2)$ **3.** $5(70 - 2)$ **4.** $9(20 - 1)$

5. $4(50 - 3)$ **6.** $6(80 + 5)$ **7.** $4 \cdot 52$ **8.** $6 \cdot 64$

9. $4(9.5)$ **10.** $6(9.9)$ **11.** $14\left(1\frac{1}{7}\right)$ **12.** $15\left(4\frac{3}{5}\right)$

Simplify.

13. $30 \cdot 18 + 70 \cdot 18$
14. $11 \cdot 43 + 89 \cdot 43$
15. $(13 \cdot 27) - (13 \cdot 27)$
16. $(64 \cdot 81) + (36 \cdot 81)$
17. $(0.75)(32) + (0.25)(32)$
18. $(3.6)(25) - (1.6)(25)$
19. $5a + 2a$
20. $7x + 6x$
21. $13y - 4y$
22. $2n + (-8)n$
23. $(-3)p + 7p$
24. $(-8)n - 7n$

For each expression write an equivalent expression without parentheses.

25. $3(x + 2)$
26. $5(a + 7)$
27. $4(n - 2)$
28. $8(b - 5)$
29. $6(3n + 2)$
30. $7(5n + 3)$
31. $3(j - k)$
32. $2(3x - y)$
33. $(3n + 7)2$
34. $(4x + 3)3$
35. $(2x + 3y)5$
36. $(6m + 7n)2$

Simplify.

37. $3a + 7 + 5a$
38. $6n + 1 + 3n$
39. $8n - 5 + 4n$
40. $3p + 8 - 3p$
41. $8y - 7y + 4$
42. $y + 9z - 6z$

Sample $8x + y + 2x + 6y = (8x + 2x) + (y + 6y)$
$= 10x + 7y$ ***Answer***

43. $2a + b + 5a + 3b$
44. $4c + d + 3c + 2d$
45. $3s + 5t - t - 2s$
46. $5d - 3e + 4e - d$
47. $7x + 5y - x - 5y$
48. $6g - 3f + 8f - g$

B

49. $4(x + 3) + 3$
50. $3(n - 1) + 4$
51. $3(a - 7) - 4$
52. $2(n - 8) + 9$
53. $7 + 2(5 - x)$
54. $8 + 3(4 - y)$
55. $2 + (x + 3)5$
56. $(y - 5)6 + 15$
57. $5(y + 3) + 7y$
58. $9(n - 3) + 4n$
59. $3(2x + y + 1)$
60. $5(7y - 3z + 4)$
61. $9(a + b) + 4(3a + 2b)$
62. $8(k + m) + 15(2k + 5m)$
63. $6(r + 5) + 9(r - 2) - 4r$
64. $4(n + 7) + 5(n - 3) - 2n$
65. $7(c + 2d + 8) + 3(9c - 2)$
66. $4(5x + 3y + 6) + 14(2y - 1)$

In Exercises 67–70, represent each word phrase by a variable expression. Then simplify it.

67. Five times the sum of c and d, increased by twice the sum of $3c$ and d
68. Twice the sum of eleven and x, increased by three times the difference between x and seven
69. Eight more than the sum of -5 and $15y$, increased by one half of the difference between $12y$ and 8
70. Six more than three times the sum of a and b, increased by five less than b

Use the 5-step plan to solve each problem over the given domain.

C **71.** If a number is increased by 17 and this sum is multiplied by 4, the result is 77 more than the number. Find the number.
Domain: $\{2, 3, 4\}$

72. If five is subtracted from twice a number and this difference is tripled, the result is one more than four times the number. Find the number.
Domain: $\{8, 9, 10\}$

Simplify.

73. $8[5x + 7(3 + 4x)] - 16x - 8$

74. $-24 + 3[4y + 2(5y - 8)] - 9y$

75. $12(3n + 2p) + 11[n + 3(2n - p - 3)]$

76. $9[7(3a + 2b - 4) + 12(a - 2)] + 3(5a - 8b)$

77. $3[4y - 2(x + 3) + (2 - y)]$

78. $9[4(2n + r) - 11r] + 3(r - n)$

Mixed Review Exercises

Evaluate if $a = -3$, $b = -2$, $c = 2$, $x = 4$, and $y = 5$.

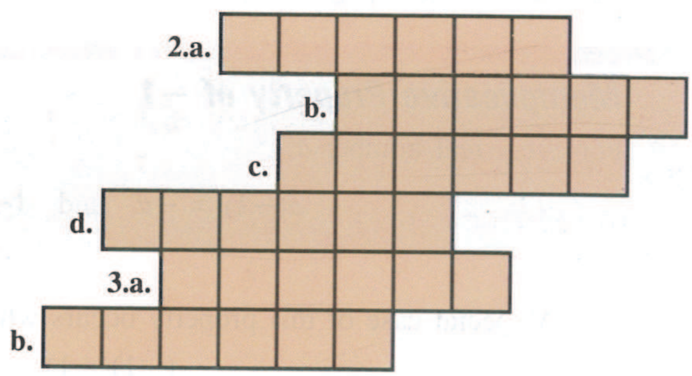
1. $5x + 2y - c$
2. $(c \cdot c + x) \div 2$
3. $2y - (4x \div c)$
4. $|a| + |b| + (-c)$
5. $c + |-y| + |b|$
6. $3|a| - |b|$
7. $-(-b + x) + y$
8. $b + y + (-2)$
9. $a + x + (-y)$
10. $x - (a - b)$
11. $-a - b + y$
12. $|b + a| - y$

Challenge

1. Copy the diagram.

2. Find each product and write it in the appropriate box. A calculator may be helpful.

a. $142{,}857 \cdot 1$
b. $142{,}857 \cdot 2$
c. $142{,}857 \cdot 3$
d. $142{,}857 \cdot 4$



3. Predict the following products from the pattern you see in Exercise 2.

a. $142{,}857 \cdot 5$
b. $142{,}857 \cdot 6$

2-6 Rules for Multiplication

Objective To multiply real numbers.

When you multiply any given real number by 1, the product is equal to the given number. For example:

$$4 \cdot 1 = 4 \quad \text{and} \quad 1 \cdot 4 = 4$$

The **identity element for multiplication** is 1.

Identity Property of Multiplication

There is a unique real number 1 such that for every real number a,

$$\boldsymbol{a \cdot 1 = a} \quad \text{and} \quad \boldsymbol{1 \cdot a = a.}$$

The equations $4 \cdot 0 = 0$ and $0 \cdot 4 = 0$

illustrate the *multiplicative property of zero:* When one (or at least one) of the factors of a product is zero, the product itself is zero.

Multiplicative Property of Zero

For every real number a:

$$\boldsymbol{a \cdot 0 = 0} \quad \text{and} \quad \boldsymbol{0 \cdot a = 0}$$

Would you guess that $4(-1) = -4$? You can verify this product by noticing that

$$4(-1) = (-1) + (-1) + (-1) + (-1) = -4.$$

Multiplying *any* real number by -1 produces the opposite of the number. (See Exercise 59, page 73.)

Multiplicative Property of −1

For every real number a:

$$\boldsymbol{a(-1) = -a} \quad \text{and} \quad \boldsymbol{(-1)a = -a}$$

A special case of this property occurs when the value of a is -1.

$$(-1)(-1) = 1$$

Using the multiplicative property of -1 with the familiar multiplication facts for positive numbers and properties that you have learned, you can compute the product of *any* two real numbers.

Example 1 Multiply: **a.** $4(7)$ **b.** $(-4)(7)$ **c.** $4(-7)$ **d.** $(-4)(-7)$

Solution

a. $4(7) = 28$ ***Answer***

b. $(-4)(7) = (-1)4(7)$
$= (-1)28$
$= -28$ ***Answer***

c. $4(-7) = 4(-1)(7)$
$= 4(7)(-1)$
$= 28(-1)$
$= -28$ ***Answer***

d. $(-4)(-7) = (-1)4(-1)7$
$= (-1)(-1)4(7)$
$= 1(28)$
$= 28$ ***Answer***

Property of Opposites in Products

For all real numbers a and b:

$$(-a)(b) = -ab \qquad a(-b) = -ab \qquad (-a)(-b) = ab$$

Practice in computing products will suggest to you the following rules for multiplication of positive and negative numbers.

Rules for Multiplication

1. If two numbers have the *same* sign, their product is *positive*. If two numbers have *opposite* signs, their product is *negative*.
2. The product of an *even* number of negative numbers is *positive*. The product of an *odd* number of negative numbers is *negative*.

Example 2

a. $4(-6)(-7)(-5)$ is negative because it has 3 negative factors.
b. $(-2)(-8)(-7)(5)(-6)$ is positive because it has 4 negative factors.
c. $(-9)(3)(0)(-5)$ is zero because it has a zero factor.

Example 3 Simplify: **a.** $(-3x)(-4y)$ **b.** $4p + (-5p)$

Solution

a. $(-3x)(-4y) = (-3)x(-4)y$
$= (-3)(-4)xy$
$= 12xy$ ***Answer***

b. $4p + (-5p) = [4 + (-5)]p$
$= (-1)p$
$= -p$ ***Answer***

Example 4 Simplify: **a.** $-2(x - 3y)$ **b.** $3p - 4(p - 2)$

Solution

a. $-2(x - 3y) = -2x - (-2)(3y)$
$= -2x - (-6y)$
$= -2x + 6y$
Answer

b. $3p - 4(p - 2) = 3p - (4p - 4 \cdot 2)$
$= 3p - (4p - 8)$
$= 3p - 4p + 8$
$= -p + 8$ ***Answer***

Oral Exercises

Simplify.

1. $(-3)(-1)$ **2.** $10(-8)$ **3.** $(-3)(-9)$ **4.** $(-8)(-5)$

5. $(-1)(6)(-6)$ **6.** $2(-6)(-1)$ **7.** $(-1)(-3)(-5)$ **8.** $(-2)(-4)(-6)$

9. $7(0)(-12)$ **10.** $(-11)(-7)(0)$ **11.** $(-6)(-10g)$ **12.** $(-7)(12p)$

13. $(-3p)(-4q)$ **14.** $(4e)(-6f)$ **15.** $(-9v)(2w)$

16. $-2ab - 3ab$ **17.** $7c + (-7c)$ **18.** $-7rs + 2rs$

19. $-5wz + (-4wz)$ **20.** $8u + (-8u)$ **21.** $-9xy + 8xy$

22. $-3(x - y)$ **23.** $-2(a - 5b)$ **24.** $-2(c + 5d)$

25. $-4(3b - 5)$ **26.** $(x - 7)(-2)$ **27.** $(-m - n)(-9)$

28. $1 - 2(g + h)$ **29.** $5 - 2(a - b)$ **30.** $-(x - 7) + 4$

Written Exercises

Multiply.

A

1. $(-37)(-2)$ **2.** $23(-5)$ **3.** $(-4)(10)(-12)$

4. $(-6)(-9)(20)$ **5.** $(-3)(-7)(-4)$ **6.** $(-2)(-8)(-4)$

7. $(-17)(-18)(0)$ **8.** $54(-47)(0)$ **9.** $5(-3)(-10)(-2)$

10. $(-4)(25)(-2)(-3)$ **11.** $(-6)(-1)(-7)(-10)$ **12.** $(-9)(-5)(-1)(-3)$

13. $(-2a)(-3b)$ **14.** $(-5x)(-3y)$ **15.** $2p(-7q)$

16. $(-4e)(5t)$ **17.** $(-7a)(-5c)4$ **18.** $(-x)(-3y)(-z)$

19. $-2(x - 3y)$ **20.** $-5(c + 2d)$ **21.** $-7(3m + 4n)$

22. $-9(-5y - 8)$ **23.** $(5x - 3)(-7)$ **24.** $(-4 + 7y)(-2)$

25. $(-1)(a + b - 3)$ **26.** $(-1)(2n - y - 5)$ **27.** $(-1)(a - b - c)$

28. $(-1)(-x - y + 7)$ **29.** $-[(-2)(a + b)]$ **30.** $-[(-3)(x - y)]$

Simplify.

31. $5x - 7x + 8 + 2x$ **32.** $3y - 7 - 8y + 5$

33. $14p - 7c - 9c + 11p$ **34.** $-y - 3z + 4y - 9z - y$

B

35. $3.4y + 1.6s - (-1.9y) - 3.6s$ **36.** $-0.8c + 4.1h - (-3.2c) - 0.1h$

37. $-8x - 7x - 5x$ **38.** $-34n + 18n - 4n$

39. $-8(-19) - 7(-19) - 5(-19)$ **40.** $-34 \cdot 9 + 18 \cdot 9 - 4 \cdot 9$

41. $88(-57) + 13(-57) + (-1)(-57)$ **42.** $-63 \cdot 81 + 56 \cdot 81 - 13 \cdot 81$

43. $-m + \frac{2}{3}m - \frac{1}{2}n + \frac{5}{2}n$ **44.** $-5a - \left(-\frac{1}{2}b\right) + 3\frac{1}{2}a - \frac{1}{2}b$

45. $7 - 3(r + s)$

46. $2 + 6(m - n)$

47. $2(x + 5y) + (-3)(7x - y)$

48. $8(t - u) + 5(2t - 3u)$

49. $-2(2q + w) - 7(w - q)$

50. $-3(7c + d) - 2(10d - c)$

51. $-4(-e + 3f) - 3[e + (-5f)]$

52. $-6[v + (-9w)] + (-5)(3v - w)$

C 53. $2[-7(r + 2s) - r] - 3(s + 2r)$

54. $4[2(-5x + y) - y] - 10(y - 4x)$

55. $-15 + (-3)[2(g - 7) - 2(1 - g)]$

56. $-50 + (-2)[3(1 - f) - 3(-2 + f)]$

Write your answer as a variable expression.

57. Sal owned 500 shares of Acme Tube. On Monday morning each share of the stock had gained p points. On Tuesday each share lost one more than twice as many points as it had gained the day before. How much did the total value of Sal's shares of Acme Tube change between the opening of trading on Monday and the closing on Tuesday?

58. A discount store bought 12 dozen radios, each to be sold at \$15 above cost. The store sold x of the radios at that price. Each of the remaining radios was sold at \$4 below cost. If the store paid \$11 for each radio, what was the total income from the sale? (*Hint:* Income = Sales − Cost)

59. To show that $a(-1)$ is the opposite of a for every real number a, you can show that the sum of $a(-1)$ and a is zero as follows. Name the property that justifies each step.

$a(-1) + a = a(-1) + a(1)$ **a.** _?_
$= a[(-1) + 1]$ **b.** _?_
$= a(0)$ **c.** _?_
$= 0$ **d.** _?_

Since the unique opposite of a is $-a$, $a(-1) = -a$.

60. Name the property or definition that justifies each step.

$a(b - c) = a[b + (-c)]$ **a.** _?_
$= a(b) + a(-c)$ **b.** _?_
$= ab + (-ac)$ **c.** _?_
$= ab - ac$ **d.** _?_

Mixed Review Exercises

Translate each sentence into an equation.

1. Four times a number is 44.
2. The sum of n and 2 is 32.
3. Half of a number is twelve.
4. Seven more than twice a number is 11.

Simplify.

5. $120 - (14 - 6)$
6. $191 - (9 - 12)$
7. $3 + (-2) + (-y) + 11$
8. $4(30 + 2)$
9. $3n + (-7n)$
10. $4(n + 2) + 8$

Application / *Understanding Product Prices*

You may have noticed the Universal Product Code (UPC) on items in your supermarket. This code is a series of bands of alternating light and dark spaces of varying widths that represent the numbers printed underneath. Many supermarkets have installed electronic check-out counters that contain scanners. These scanners read the UPC on items and send the information to a central computer. The computer "looks up" the price of the item and subtracts the item from the store's inventory of products. The product name and price are then printed on the sales slip. The customer benefits by having a record of the transaction and the store benefits by having up-to-date inventory records.

0 39800 00320

Manufacturer Code | Product Code

Consumers must make many choices while shopping. Quality, price, and convenience are all important considerations. Sometimes it is difficult to compare the prices of products in different-sized packages. Unit pricing can help you make the comparison. The unit price of an item is its price per unit of measure. Unit prices are often posted on the shelves with the products. The product with the lowest unit price is the best buy provided the quality and the quantity meet your needs.

Example Find the unit price per liter of lowfat milk: \$.95 for 950 mL.

Solution $1 \text{ L} = 1000 \text{ mL}$, so $950 \text{ mL} = \frac{950}{1000} \text{ L} = 0.95 \text{ L}$.

$\$.95 \div 0.95 \text{ L} = \$1.00$ per liter

Exercises

Find the unit price per liter of each item. A calculator may be helpful.

1. Shampoo: \$2.40 for 480 mL **2.** Chicken noodle soup: \$.63 for 315 mL

3. Soy sauce: \$1.98 for 600 mL **4.** Grapefruit juice: \$1.44 for 1440 mL

2-7 Problem Solving: Consecutive Integers

Objective To write equations to represent relationships among integers.

When you want to be served at a bakery, a deli, or a pizza parlor you often must take a number. The numbers help the clerks keep track of who is next in line.

When you count by ones from any number in the set of integers,

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\},$$

you obtain **consecutive integers.** For example, -2, -1, 0, 1, and 2 are five consecutive integers. Those integers are listed in *natural order,* that is, in order from least to greatest.

Example 1 An integer is represented by n.

a. Write the next three integers in natural order after n.
b. Write the integer that immediately precedes n.
c. Write an equation that states that the sum of four consecutive integers, starting with n, is 66.

Solution

a. $n + 1, n + 2, n + 3$
b. $n - 1$
c. $n + (n + 1) + (n + 2) + (n + 3) = 66$

Ten is called an *even* integer because $10 = 2 \cdot 5$. An integer that is the product of 2 and any integer is an **even integer.** In natural order, they are

$$\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots$$

An **odd integer** is an integer that is not even. In natural order, the odd integers are

$$\ldots, -5, -3, -1, 1, 3, 5, \ldots$$

If you count by *twos* beginning with any even integer, you obtain **consecutive even integers.** For example:

Three consecutive even integers: 8; $8 + 2$, or 10; $10 + 2$, or 12

If you count by *twos* beginning with any odd integer, you obtain **consecutive odd integers.** For example:

Three consecutive odd integers: 9; $9 + 2$, or 11; $11 + 2$, or 13

Thus, in natural order, $n, n + 2, n + 4$

are said to be consecutive even integers if n is even, and consecutive odd integers if n is odd.

Example 2 Solve over the given domain: The sum of three consecutive odd integers is 100 more than the smallest integer. What are the integers?
Domain for the smallest integer: {27, 37, 47}

Solution

Step 1 The unknowns are the 3 consecutive odd integers.

Step 2 Let n = the smallest integer, $n + 2$ = the middle integer, and $n + 4$ = the largest integer.

Step 3 $\underbrace{n + (n + 2) + (n + 4)}_{\text{The sum}} \overset{\text{is}}{=} \underbrace{n + 100}_{\text{100 more than the smallest integer.}}$

Step 4

n	$n + (n + 2) + (n + 4) = n + 100$	
27	27 + 29 + 31 = 27 + 100	False
37	37 + 39 + 41 = 37 + 100	False
47	47 + 49 + 51 = 47 + 100	True

Step 5 Are 47, 49, and 51 consecutive odd integers whose sum is 147?
The check is left to you. ∴ the integers are 47, 49, and 51. ***Answer***

Oral Exercises

1. If $w = 3$, what are the values of $w + 1$, $w + 2$, and $w + 3$?

2. If $r = 1$, what are the values of $r - 1$, $r - 2$, and $r - 3$?

3. The smallest of four consecutive integers is -1. What are the other three integers?

4. The greatest of four consecutive integers is 12. What are the other three integers?

5. If $x = 15$, represent 14 and 16 in terms of x.

6. If $p = 20$, represent 18 and 22 in terms of p.

7. If $k = 11$, represent 7, 9, and 13 in terms of k.

8. If $e = 12$, represent 8, 10, 14, and 16 in terms of e.

9. If m is an odd integer, is $m + 1$ odd or is it even? $m + 2$? $m - 1$?

10. If m and n are even, is $m + n$ odd or is it even? $m - n$? mn?

11. If m and n are odd, is $m + n$ odd or is it even? $m - n$? mn?

12. If m is odd and n is even, is $m + n$ odd or is it even? $m - n$? mn?

13. If n is an integer, is $2n$ odd or even? What are the next two even integers greater than $2n$? What is the next smaller even integer?

14. If n is an integer, is $2n + 1$ odd or even? What are the next two odd integers greater than $2n + 1$? What is the next smaller odd integer?

Written Exercises

Write an equation to represent the given relationship among integers.

Sample The product of two consecutive integers is 8 more than twice their sum.

Solution Let x = the first of the integers.
Then $x + 1$ = the second integer.

Their product is 8 more than twice their sum.

$$\underbrace{x(x+1)}_{\text{Their product}} \; = \; \underbrace{2[x + (x + 1)] + 8}_{\text{8 more than twice their sum}}$$

$\therefore$ the equation is $x(x + 1) = 2[x + (x + 1)] + 8$. ***Answer***

A

1. The sum of two consecutive integers is 43.
2. The sum of three consecutive integers is 69.
3. The sum of four consecutive integers is -106.
4. The sum of four consecutive integers is -42.
5. The sum of three consecutive odd integers is 75.
6. The sum of three consecutive odd integers is 147.
7. The product of two consecutive even integers is 168.
8. The product of two consecutive odd integers is 195.
9. The sum of four consecutive even integers is -100.
10. The greater of two consecutive even integers is six less than twice the smaller.
11. Four cousins were born at two-year intervals. The sum of their ages is 36.
12. The smaller of two consecutive even integers is five more than one half of the greater.

Solve each problem over the given domain.

B

13. The sum of three consecutive odd integers is 40 more than the smallest. What are the integers?
Domain for the smallest: $\{13, 17, 25\}$
14. The sum of three consecutive even integers is 30 more than the largest. What are the integers?
Domain for the smallest: $\{14, 18, 20\}$
15. Find two consecutive integers whose product is 5 less than 5 times their sum.
Domain for the smallest: $\{0, 6, 9\}$
16. Find two consecutive odd integers whose product is 1 less than 6 times their sum.
Domain for the smallest: $\{-1, 1, 11\}$

Write an equation to represent the given relationship among integers. Use this definition:

The product of any real number and an integer is called a ***multiple*** *of the real number.*

C **17.** The lengths in feet of three ropes are consecutive multiples of 3. If each rope were 4 ft shorter, the sum of their lengths would be 42 ft.

18. Jim weighs more than Joe but less than Jack. Their weights in kilograms are consecutive multiples of 7. If they each weighed 5 kg less, the sum of their weights would be 195 kg.

Mixed Review Exercises

Simplify.

1. $(40 - 7) - (55 - 20)$ **2.** $-5.3 + 2.1 - 1.7$ **3.** $-4 + 2c + (-1) + 8$

4. $\frac{6}{5} + \left(-\frac{9}{5}\right)$ **5.** $3\frac{1}{3} + 12 + 2\frac{2}{3}$ **6.** $6\left(\frac{2}{5}\right) - 5\left(\frac{2}{5}\right) + 3\left(\frac{1}{5}\right)$

7. $-(11 - x) - (x - 13)$ **8.** $15a - 3a + 7a$ **9.** $12 + 5y + 6 + (-2)$

10. $(-1)(x + y - z)$ **11.** $-[-6(a - b)]$ **12.** $3(-5 + y)$

Self-Test 2

Vocabulary

distributive property (p. 65)
equivalent expressions (p. 67)
simplify a variable expression (p. 67)
identity element for multiplication (p. 70)
identity property of multiplication (p. 70)
multiplicative property of zero (p. 70)
multiplicative property of -1 (p. 70)
property of opposites in products (p. 71)
consecutive integers (p. 75)
natural order (p. 75)
even integer (p. 75)
odd integer (p. 75)
consecutive even integers (p. 75)
consecutive odd integers (p. 75)

Simplify.

1. $x + 2y + 3x + 4y$ **2.** $4(y - 2) + 1$ **Obj. 2-5, p. 65**

3. $(-2)a + 4a$ **4.** $2(x + 3) - 2$

5. $(-5)(3)(-6)(-2)$ **6.** $(-4x)(-2y)(7)$ **Obj. 2-6, p. 70**

7. $-3(a + b - c)$ **8.** $18 \cdot 12 + 18 \cdot 10$

9. If $x = 30$, represent 29 and 31 in terms of x. **Obj. 2-7, p. 75**

10. Write an equation to represent the given relationship among the integers: The greater of two consecutive odd integers is one less than twice the smaller.

Check your answers with those at the back of the book.

Division

2-8 The Reciprocal of a Real Number

Objective To simplify expressions involving reciprocals.

Two numbers whose product is 1 are called **reciprocals,** or **multiplicative inverses,** of each other. For example:

1. 5 and $\frac{1}{5}$ are reciprocals because $5 \cdot \frac{1}{5} = 1$.
2. $\frac{4}{5}$ and $\frac{5}{4}$ are reciprocals because $\frac{4}{5} \cdot \frac{5}{4} = 1$.
3. -1.25 and -0.8 are reciprocals because $(-1.25)(-0.8) = 1$.
4. 1 is its own reciprocal because $1 \cdot 1 = 1$.
5. -1 is its own reciprocal because $(-1)(-1) = 1$.
6. 0 has no reciprocal because 0 times *any* number is 0, *not* 1.

The symbol for the reciprocal, or multiplicative inverse, of a nonzero real number a is $\frac{1}{a}$. Every real number except 0 has a reciprocal.

Property of Reciprocals

For every *nonzero* real number a, there is a unique real number $\frac{1}{a}$ such that

$$\boldsymbol{a \cdot \frac{1}{a} = 1 \quad \text{and} \quad \frac{1}{a} \cdot a = 1.}$$

Look at the following product:

$$(-a)\left(-\frac{1}{a}\right) = (-1a)\left(-1 \cdot \frac{1}{a}\right) = (-1)(-1)\left(a \cdot \frac{1}{a}\right) = 1 \cdot 1 = 1$$

Therefore, $-a$ and $-\frac{1}{a}$ are reciprocals.

Property of the Reciprocal of the Opposite of a Number

For every *nonzero* number a,

$$\boldsymbol{\frac{1}{-a} = -\frac{1}{a}.}$$

Read, "The reciprocal of $-a$ is $-\frac{1}{a}$."

Now look at this product:

$$(ab)\left(\frac{1}{a} \cdot \frac{1}{b}\right) = \left(a \cdot \frac{1}{a}\right)\left(b \cdot \frac{1}{b}\right) = 1 \cdot 1 = 1$$

Therefore, ab and $\frac{1}{a} \cdot \frac{1}{b}$ are reciprocals.

Property of the Reciprocal of a Product

For all *nonzero* numbers a and b,

$$\frac{1}{ab} = \frac{1}{a} \cdot \frac{1}{b}.$$

The reciprocal of the product of two nonzero numbers is the product of their reciprocals.

Example 1 Simplify: **a.** $\frac{1}{4} \cdot \frac{1}{-7}$ **b.** $4y \cdot \frac{1}{4}$ **c.** $(-6ab)\left(-\frac{1}{3}\right)$

Solution **a.** $\frac{1}{4} \cdot \frac{1}{-7} = \frac{1}{4(-7)} = \frac{1}{-28} = -\frac{1}{28}$ ***Answer***

b. $4y \cdot \frac{1}{4} = \left(4 \cdot \frac{1}{4}\right)y = 1y = y$ ***Answer***

c. $(-6ab)\left(-\frac{1}{3}\right) = (-6)\left(-\frac{1}{3}\right)(ab) = 2ab$ ***Answer***

Example 2 Simplify $\frac{1}{3}(42m - 3v)$.

Solution

$$\begin{aligned}\frac{1}{3}(42m - 3v) &= \frac{1}{3}(42m) - \frac{1}{3}(3v)\\ &= \left(\frac{1}{3} \cdot 42\right)m - \left(\frac{1}{3} \cdot 3\right)v\\ &= 14m - v\end{aligned}$$ ***Answer***

Oral Exercises

State the reciprocal in simplest form.

1. 7 **2.** 1 **3.** -1 **4.** $\frac{1}{11}$

5. -2 **6.** $\frac{3}{4}$ **7.** 0.25 **8.** $-\frac{1}{8}$

9. $-\frac{13}{5}$ **10.** $-\frac{2}{3}$ **11.** $\frac{1}{w}$, $w \neq 0$ **12.** $-\frac{1}{s}$, $s \neq 0$

Simplify.

13. $\frac{1}{6} \cdot \frac{1}{10}$
14. $\frac{1}{-2} \cdot \frac{1}{-12}$
15. $\frac{1}{6} \cdot \frac{1}{-4}$
16. $\frac{1}{-x} \cdot \frac{1}{y}$, $x \neq 0$, $y \neq 0$
17. $(3a)\frac{1}{3}$
18. $\frac{1}{5}(5x)$
19. $6w \cdot \frac{1}{6}$
20. $\frac{1}{-3}(3ab)$
21. $\frac{1}{2}(10z + 12)$
22. $\frac{1}{7}(21g - 14)$
23. $-\frac{1}{4}(16m + 32)$
24. $-\frac{1}{6}(54t - 18)$
25. **a.** $\frac{1}{\frac{2}{3}}$ represents the reciprocal of _?_. **b.** In simplest form $\frac{1}{\frac{2}{3}} =$ _?_.

Written Exercises

Simplify each expression.

A

1. $\frac{1}{5}(-20)$
2. $-\frac{1}{12}(48)$
3. $-1000\left(\frac{1}{100}\right)$
4. $-70\left(\frac{1}{7}\right)$
5. $96\left(-\frac{1}{8}\right)\left(-\frac{1}{12}\right)$
6. $-63\left(-\frac{1}{3}\right)\left(-\frac{1}{21}\right)$
7. $\frac{1}{-3}(36)\left(\frac{1}{4}\right)$
8. $-150\left(\frac{1}{2}\right)\left(-\frac{1}{3}\right)$
9. $4r\left(-\frac{1}{4}\right)$
10. $33p\left(-\frac{1}{11}\right)$
11. $\frac{1}{x}(5x)$, $x \neq 0$
12. $7t\left(\frac{1}{t}\right)$, $t \neq 0$
13. $12xy\left(\frac{1}{3}\right)$
14. $15mn\left(\frac{1}{5}\right)$
15. $8ab\left(-\frac{1}{4}\right)$
16. $9cd\left(\frac{1}{-3}\right)$
17. $(-4pg)\left(\frac{1}{-2}\right)$
18. $(-36ac)\left(\frac{1}{-9}\right)$

B

19. $\frac{1}{2}(-16a + 20)$
20. $\frac{1}{3}(18b - 39)$
21. $-\frac{1}{5}(-45c + 10d)$
22. $-\frac{1}{8}(56g - 72h)$
23. $(-42m - 91k)\left(-\frac{1}{7}\right)$
24. $(-39n - 52p)\left(-\frac{1}{13}\right)$
25. $\frac{1}{2}(8u + 10v) - \frac{1}{3}(15u - 3v)$
26. $\frac{1}{5}(-5a + 20b) - \frac{1}{2}(2b - 6a)$
27. $6\left(\frac{1}{3}x - \frac{1}{2}y\right) + 42\left(-\frac{1}{3}y - \frac{1}{7}x\right)$
28. $-8\left(-\frac{1}{8}p + q\right) + \frac{1}{9}(63p - 9q)$
29. $-\frac{1}{8}(48m - 16) - \frac{1}{4}(84m + 8)$
30. $-5\left(4 - \frac{1}{2}n\right) + \frac{1}{16}(-32n + 8)$

C

31. $-\frac{1}{12}(6r + 4s) + 7\left(\frac{1}{21}s - \frac{1}{14}r\right)$
32. $-\frac{1}{20}(5z - 4w) - 6\left(-\frac{1}{30}w - \frac{1}{24}z\right)$
33. $-3\left[\frac{1}{4}(12n + 1) - \frac{1}{4}\right] + 10n$
34. $3s + \left(-\frac{1}{2}\right)\left[6 + 24\left(-\frac{1}{3} + \frac{1}{4}s\right)\right]$

Use the five-step plan for solving each problem over the given domain.

35. The sum of a number and its reciprocal is $\frac{13}{6}$. Find the number.

Domain: $\left\{\frac{2}{3}, \frac{3}{4}, \frac{5}{4}, \frac{3}{2}\right\}$

36. A number is $2\frac{1}{10}$ more than its reciprocal. Find the number.

Domain: $\left\{\frac{2}{5}, \frac{3}{5}, \frac{5}{4}, \frac{5}{2}\right\}$

Mixed Review Exercises

Translate each sentence into an equation.

1. Four more than five times a number is 24.

2. Fifteen less than a number is 250.

3. The sum of two consecutive integers is 67.

4. The product of two consecutive integers is 42.

Simplify.

5. $(-12)(-5)(-2)$

6. $-36(25)(-2)$

7. $(3)(-7)(-2)$

8. $-7(5a - 2d)$

9. $-4(2 + x) - 3(x - 2)$

10. $8(x - 1) + 5(2 - x)$

Calculator Key-In

Use the reciprocal key on a calculator to find the reciprocal of each number.

1. 0.0625

2. −32

3. 3125

4. 0.000064

5. For each number in Exercises 1–4, press the reciprocal key twice. Your results illustrate the property: The reciprocal of the reciprocal of a number is __?__.

6. a. Copy and complete the table.

a	b	$\frac{1}{ab}$	$\frac{1}{a} \cdot \frac{1}{b}$
4	16	?	?
−32	−0.5	?	?
0.234	0.654	?	?
555	222	?	?

b. What property does your completed table illustrate?

2-9 Dividing Real Numbers

Objective To divide real numbers and to simplify expressions involving quotients.

Dividing by 2 is the same as multiplying by $\frac{1}{2}$: $8 \div 2 = 4$ and $8 \cdot \frac{1}{2} = 4$.

Dividing by 5 is the same as multiplying by $\frac{1}{5}$: $15 \div 5 = 3$ and $15 \cdot \frac{1}{5} = 3$.

The examples above illustrate how division is related to multiplication.

Definition of Division

For every real number a and every *nonzero* real number b, the **quotient** $a \div b$, or $\frac{a}{b}$, is defined by:

$$a \div b = a \cdot \frac{1}{b}.$$

To divide by a nonzero number, multiply by its reciprocal.

You can use the definition of division to express any quotient as a product.

Example 1 a. $\frac{24}{6} = 24 \div 6 = 24 \cdot \frac{1}{6} = 4$

b. $\frac{24}{-6} = 24 \div (-6) = 24\left(-\frac{1}{6}\right) = -4$

c. $\frac{-24}{6} = -24 \div 6 = -24 \cdot \frac{1}{6} = -4$

d. $\frac{-24}{-6} = -24 \div (-6) = (-24)\left(-\frac{1}{6}\right) = 4$

The four quotients in Example 1 illustrate the following rules.

Rules for Division

If two numbers have the *same* sign, their quotient is *positive*.
If two numbers have *opposite* signs, their quotient is *negative*.

Example 2 a. $(-9) \div \left(-\frac{10}{3}\right) = (-9)\left(-\frac{3}{10}\right) = \frac{27}{10}$ ***Answer***

b. $\frac{-3}{-\frac{1}{3}} = (-3) \div \left(-\frac{1}{3}\right) = (-3)(-3) = 9$ ***Answer***

Example 3 $\frac{45x}{-9} = 45x\left(-\frac{1}{9}\right) = 45\left(-\frac{1}{9}\right)x = -5x$ ***Answer***

Example 4 $\frac{w}{13} \cdot 13 = w \cdot \frac{1}{13} \cdot 13 = w \cdot 1 = w$ ***Answer***

Here are some important questions and answers about division of real numbers:

1. Why can you never divide by zero? Dividing by 0 would mean multiplying by the reciprocal of 0. But 0 has no reciprocal (page 79). Therefore, *division by zero has no meaning in the set of real numbers.*

2. Can you divide zero by any number other than zero? Yes, for example,

$$\frac{0}{5} = 0 \cdot \frac{1}{5} = 0 \quad \text{and} \quad 0 \div (-2) = 0 \cdot \left(-\frac{1}{2}\right) = 0.$$

When zero is divided by any nonzero number, the quotient is zero.

3. Is division commutative? No, for example,

$$8 \div 2 = 4 \quad \text{but} \quad 2 \div 8 = 0.25.$$

4. Is division associative? No, for example,

$$(12 \div 3) \div 2 = 4 \div 2 = 2 \quad \text{but} \quad 12 \div (3 \div 2) = 12 \div 1.5 = 8.$$

The following properties of division will be proved in Exercises 34 and 35 on page 86.

For all real numbers a, b, and c such that $c \neq 0$,

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad \text{and} \quad \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}.$$

Oral Exercises

Read each quotient as a product. Then simplify.

Sample $8 \div \left(-\frac{1}{6}\right)$ **Solution** Eight times negative six; -48

1. $\frac{-21}{3}$
2. $\frac{0}{8}$
3. $-25 \div 5$
4. $18 \div (-18)$
5. $\frac{0}{-9}$
6. $\frac{-6}{-6}$
7. $\frac{-2}{2}$
8. $\frac{49}{-7}$
9. $\frac{38}{-2}$
10. $\frac{-56}{-8}$
11. $\frac{4}{-64}$
12. $\frac{-8}{72}$

Simplify.

13. $x \div 1$
14. $x \div (-1)$
15. $8a \div (-2)$
16. $(-18b) \div 3$
17. $(-60a) \div (-5)$
18. $(-48b) \div (-6)$
19. $\frac{a}{a}$, $a \neq 0$
20. $\frac{-a}{a}$, $a \neq 0$
21. $6 \cdot \frac{n}{6}$
22. $\left(-\frac{s}{4}\right)(-4)$
23. $2\left(\frac{y}{-2}\right)$
24. $-3 \cdot \frac{n}{3}$

Written Exercises

Simplify.

A

1. $-48 \div 6$
2. $-64 \div 4$
3. $-12 \div (-3)$
4. $-24 \div (-8)$
5. $8 \div \left(-\frac{1}{2}\right)$
6. $12 \div \left(-\frac{1}{3}\right)$
7. $0 \div \frac{3}{4}$
8. $-6 \div \left(-\frac{1}{4}\right)$
9. $\frac{-6}{-\frac{1}{6}}$
10. $\frac{-8}{-\frac{1}{3}}$
11. $\frac{-7}{\frac{1}{5}}$
12. $\frac{9}{-\frac{1}{9}}$
13. $3 \cdot \frac{x}{3}$
14. $\frac{-w}{4} \cdot 4$
15. $(-4)\left(\frac{-y}{2}\right)$
16. $(-8)\left(\frac{x}{-2}\right)$
17. $\frac{156a}{12}$
18. $\frac{384b}{-8}$
19. $\frac{-54x}{6}$
20. $\frac{-144y}{-24}$

In Exercises 21–24, find the average of the given numbers. (The *average* is the sum of the numbers divided by the number of numbers.)

Sample 1 15, −3, −14, −2

Solution $\frac{15 + (-3) + (-14) + (-2)}{4} = \frac{-4}{4} = -1$ ***Answer***

21. −12, 4, −11, −7
22. 18, −21, −7, 2
23. 18, −17, −22, 16, 0
24. 18, −17, −22, 16

Evaluate each expression if $a = -3$, $b = -1$, $c = 2$, and $d = 6$.

Sample 2 $\frac{abd}{a + c} = \frac{(-3)(-1)(6)}{-3 + 2} = \frac{3(6)}{-1} = \frac{18}{-1} = -18$ ***Answer***

B

25. $\frac{b + 5c}{3 - d}$
26. $\frac{bcd}{(1 - b)a}$
27. $\frac{abc}{(1 - c)d}$
28. $\frac{7a + d}{a + c}$
29. $\frac{a - 2d}{b + 3c}$
30. $\frac{4a - d}{abc}$
31. $\frac{a - 3b}{bcd}$
32. $\frac{abc - d}{abc}$
33. $\frac{(c + a)(c - a)}{b + d}$

In Exercises 34 and 35, assume that a, b, and c are any real numbers and $c \neq 0$.

C **34.** Name the property or definition that justifies each step.

$$\frac{a+b}{c} = (a+b)\cdot\frac{1}{c} \qquad \textbf{a. } \underline{\ ?\ }$$
$$= \left(a\cdot\frac{1}{c}\right) + \left(b\cdot\frac{1}{c}\right) \qquad \textbf{b. } \underline{\ ?\ }$$
$$= \frac{a}{c} + \frac{b}{c} \qquad \textbf{c. } \underline{\ ?\ }$$

35. Show that $\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$. Use the fact that multiplication is distributive with respect to subtraction. (*Hint:* See Exercise 34.)

Mixed Review Exercises

Solve if $x \in \{0, 1, 2, 3, 4, 5, 6\}$.

1. $x + 3 = 8$ **2.** $\frac{1}{3}x = 2$ **3.** $x - 1 = 2$

4. $3x = 6$ **5.** $2x + 1 = 5$ **6.** $x \div 4 = 1$

Solve over the domain $\{0, 1, 2, 3, 4, 5\}$.

7. $\frac{1}{5}n = 1$ **8.** $5y - 1 = 14$ **9.** $x + 2 = 5$

10. $2x = 4$ **11.** $x \cdot x \cdot x = 1$ **12.** $5n = n \cdot 5$

Self-Test 3

Vocabulary reciprocals (p. 79)
multiplicative inverse (p. 79)
property of reciprocals (p. 79)
property of the reciprocal of the opposite of a number (p. 79)
property of the reciprocal of a product (p. 80)
division (p. 83)
quotient (p. 83)

State the reciprocal of each expression or number. **Obj. 2-8, p. 79**

1. $3y,\ y \neq 0$ **2.** $-\frac{1}{2}$

Simplify.

3. $-30\left(\frac{1}{3}\right)$ **4.** $\frac{1}{6}(24)(-12)$

5. $-81 \div 3$ **6.** $-36 \div (-2)$ **Obj. 2-9, p. 83**

7. $4 \div \frac{1}{2}$ **8.** $\frac{-48x}{12}$

Check your answers with those in the back of the book.

Computer Key-In

The program below will find the sum of a list of numbers that does not include zero.

```
10 PRINT "TO FIND THE SUM OF SEVERAL"
20 PRINT "NUMBERS (<>0)."
30 PRINT "(TO END, TYPE 0.)"
40 LET S=0
50 PRINT "NUMBER";
60 INPUT N
70 IF N=0 THEN 100
80 LET S=S+N
90 GOTO 50
100 PRINT "SUM=";S
110 END
```

Line 80 is not an equation. It adds each new value of N to S. It means “Take the value of S, add the value of N to it, and then put the new value into S.”

Lines 50–90 form a *loop*. Line 90 sends the program back to line 50 for the next value of N to be INPUT. Lines 50–90 will be repeated until line 70 ends the INPUT when 0 is entered. After all of the numbers in the list are INPUT, line 100 prints the final value of S.

Exercises

Use the program above to find the sum of the numbers in each list.

1. 2, −4, 6, −8, 10, −12

2. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

3. 2.25, 3.42, 5.15, 1.98, 4.82

4. 12.95, 27.59, 21.76, 38.25, 47.34

Add the three lines below to your program so that it will compute the average of the numbers in the list. The variable C acts as a counter. (You can then type LIST to get a clean copy of the modified program.)

```
45 LET C=0
85 LET C=C+1
105 PRINT "AVERAGE=";S/C
```

5–8. Find the average of the numbers in each list in Exercises 1–4.

Historical Note / *Why We Call It “Algebra”*

The word “algebra” comes from the title of a ninth-century mathematical book by the mathematician and astronomer Muhammed ibn-Musa al-Khwarizmi. The book, *hisab al-jabr w' al muqabalah* (“the science of reduction and comparison”), deals with solving equations. While the entire work may not have been original, it was the first time that algebra was systematically discussed as a separate branch of mathematics.

Al-Khwarizmi's book made its way into Europe and was translated into Latin in the twelfth century as *Ludus algebrae et almucgrabalaeque*. The title was eventually shortened to “algebra.”

Chapter Summary

1. A number line can be used to find the sum of two real numbers.
2. Opposite and absolute values are used in the rules for adding real numbers (page 54) and multiplying real numbers (page 71).
3. Real-number properties are statements about numbers that are accepted as true and that form the basis for computation in arithmetic and in algebra. The statements in the chart below are true for all real values of each variable except as noted.
4. Useful properties about addition and multiplication:

   Property of the opposite of a sum: $-(a + b) = (-a) + (-b)$

   Multiplicative property of zero: $a \cdot 0 = 0 \cdot a = 0$

   Multiplicative property of -1: $a(-1) = (-1)a = -a$

   Property of opposites in products: $(-a)(b) = -ab$, $a(-b) = -ab$, $(-a)(-b) = ab$

   Property of the reciprocal of a product: $\frac{1}{ab} = \frac{1}{a} \cdot \frac{1}{b}$; $a \neq 0$, $b \neq 0$

5. Subtraction and division are defined as follows: $a - b = a + (-b)$

   $a \div b = \frac{a}{b} = a \cdot \frac{1}{b}$; $b \neq 0$

Properties of Real Numbers

Equality:	Reflexive property	$a = a$
	Symmetric property	If $a = b$, then $b = a$.
	Transitive property	If $a = b$ and $b = c$, then $a = c$.
	Substitution principle	If $a = b$, then b may be substituted for a in any expression.

	Addition	Multiplication
Properties of closure	$a + b$ is a unique real number.	ab is a unique real number.
Commutative properties	$a + b = b + a$	$ab = ba$
Associative properties	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
Identity properties	$a + 0 = 0 + a = a$	$a \cdot 1 = 1 \cdot a = a$
Property of opposites	$a + (-a) = (-a) + a = 0$	
Property of reciprocals		$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$, $a \neq 0$
Distributive property	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$	

Chapter Review

Give the letter of the correct answer.

1. Simplify $125 + 62 + 75 + 38$. **2-1**
a. 310 **b.** 301 **c.** 290 **d.** 300

2. Simplify $(16p)(5q)$.
a. $21pq$ **b.** $80pq$ **c.** $21p + q$ **d.** $80(p + q)$

3. Simplify $13 - 5x + (-17)$. **2-2**
a. $-22x$ **b.** $-5x - 4$ **c.** -9 **d.** $5x + 30$

4. Simplify $33 + [7 + (-12)]$
a. 52 **b.** -52 **c.** 28 **d.** -28

5. Simplify $[-6 + (-1)] + [-(-6 + 1)]$. **2-3**
a. -12 **b.** -2 **c.** 0 **d.** 2

6. Ten passengers got on an empty bus. Then 3 more passengers got on, 5 got off, and 2 more got on. How many passengers were left inside?
a. 10 **b.** 9 **c.** 8 **d.** 20

7. Simplify $x + 9 - (x - 7)$. **2-4**
a. 2 **b.** 16 **c.** $2x + 16$ **d.** $2x + 2$

8. Simplify $(23 \cdot 32) - (13 \cdot 32)$. **2-5**
a. 360 **b.** 330 **c.** 320 **d.** 640

9. Simplify $3 + 7(r - 4)$.
a. $10r - 40$ **b.** $10r - 28$ **c.** $7r - 25$ **d.** $7r - 31$

10. Simplify $(-2)(6)(-1)(8)$. **2-6**
a. 96 **b.** -48 **c.** 48 **d.** -96

11. Simplify $2(3c - 2d) - 4(c - 3d)$.
a. $2c - 14d$ **b.** $10c + 10d$ **c.** $10c - 14d$ **d.** $2c + 8d$

12. Choose the equation that represents the following: The smaller of two consecutive even integers is eight less than twice the greater. **2-7**
a. $x = 8 - 2(x + 2)$ **b.** $x = 2(x + 2) - 8$
c. $x + 2 = 8 - 2(x + 2)$ **d.** $x + 2 = 2(x + 2) - 8$

13. Simplify $(-63st)\left(-\frac{1}{9}\right)$. **2-8**
a. $7s + 1$ **b.** $-7st$ **c.** $7st$ **d.** $7s - t$

14. Simplify $12\left(\frac{1}{4}x + \frac{1}{3}\right) - \frac{1}{2}(12x - 6)$.
a. $-3x + 7$ **b.** $3x + 7$ **c.** 4 **d.** $-5x + 3$

15. Simplify $\frac{72x}{-9}$. **2-9**
a. $-648x$ **b.** $8x$ **c.** -8 **d.** $-8x$

16. Evaluate $\frac{a + 15b}{c}$ if $a = -6$, $b = 9$, and $c = -3$.
a. -43 **b.** 137 **c.** -13 **d.** -47

Chapter Test

Simplify.

1. $16 + p + q + 5$ **2.** $25 \cdot 12 \cdot 4$ **2-1**

3. $-5 + [-9 + (-9)]$ **4.** $3n - 6 + (-9)$ **2-2**

5. $\left(-1\frac{3}{5}\right) + \frac{2}{5} + 1$ **6.** $-7 + 5 - 4$ **2-3**

7. Samantha left home with \$42.51. The subway fare was \$1.20. At the station she bought a magazine for \$1.95. Lunch cost \$4.36. After work she bought a skirt on sale for \$26.00. Her subway fare home was also \$1.20. At the station Shelly gave Samantha \$5 to pay back a loan. How much money did Samantha have at the end of the day?

Simplify.

8. $(25 - 31) - (-6 + 11)$ **9.** $x - (-8) - [x + (-8)]$ **2-4**

10. Kara left home $2\frac{2}{3}$ hours before she arrived at the airport. How long had she been gone from home when she had been at the airport for $1\frac{2}{3}$ hours?

Simplify.

11. $(23)(0.25) - (7)(0.25)$ **12.** $(6x + 3)4$ **2-5**

13. $5(b - 1) + 8$ **14.** $7(2c - 4d + 6)$

15. $-16(-3)$ **16.** $(-11 + 11)19$ **2-6**

17. $(-9)(8)(-1)(-3)$ **18.** $7x - y + 2x + 3y$

19. Write an equation to represent the following relationship among integers: The sum of three consecutive even integers is 30 more than the smallest integer. **2-7**

20. State the reciprocal of -1. **21.** State the reciprocal of $\frac{5}{7}$. **2-8**

Simplify.

22. $\left(-\frac{1}{17}\right)(85)\left(\frac{1}{5}\right)$ **23.** $-\frac{1}{7}(-56m + 49n)$

24. $\dfrac{-13}{\frac{1}{6}}$ **25.** $\dfrac{343w}{-7}$ **2-9**

Cumulative Review *(Chapters 1 and 2)*

Simplify.

1. $(56 \div 7) - (26 \div 13)$
2. $\frac{12 + 72}{6 + 8}$
3. $|-12| - |-6|$
4. $-|20| \div |-4|$
5. $38 + [(-3) + 16]$
6. $3[27 \div (12 \div 4)]$
7. $2 + 10 \cdot 15 \div 5$
8. $2\frac{5}{9} + 7\frac{1}{8} + 8\frac{4}{9}$
9. $-120 - (-17)$
10. $6(2x + 3) - 3(7 - 3x)$
11. $3(x + 2y) + 5(2x + y)$
12. $(-9)(-5)(4) - 9(5)$
13. $-18 - 4 - [(-6) + 12]$
14. $\frac{1}{12}(-552xy) \div (-23)$
15. $\frac{6}{7}(4a - 3b) - \frac{3}{7}(8a - 6b)$

Evaluate each expression if $x = -3$, $y = 4$, and $z = 5$.

16. $-z + (-x + 7)$
17. $x - |y| + |z|$
18. $\frac{(x + 6)(z - y)}{(x + y)}$

Write the numbers in order from least to greatest.

19. $0, -2, 4, \frac{1}{2}, -\frac{1}{2}, -3$
20. $-5, |5|, -\frac{1}{3}, -\frac{2}{3}, |-3|$
21. Graph the numbers $-4, 4, 3, 2, -\frac{1}{2}$, and 0 on a number line.

Solve.

22. $|y| = 7$
23. $|-x| = 10$
24. $-|y| = -3$
25. $c + 12 = 0$
26. $b \cdot 1 = 9$
27. $a + 25 = 25$

Solve if $x \in \{-5, -3, 0, 3, 5\}$.

28. $4x + 2 = 14$
29. $\frac{1}{3}x - 2 = -3$
30. $6 = 2x - 4$

Translate each phrase or sentence into a variable expression or an equation.

31. Five more than the product of seven and y
32. Two less than the sum of x and the opposite of a, decreased by three times y.
33. The opposite of x is four less than eight.
34. The sum of two consecutive integers is one less than twice the greater integer.

Solve.

35. Paul got into an elevator on the fourth floor. Before he got out, the elevator went up seven floors and down eight floors. On what floor did he get out?

Maintaining Skills

Perform the indicated operations.

Sample 1

$$\begin{array}{r} \overset{8\,11}{\cancel{9}\cancel{1}}7.\overset{2\,18}{\cancel{3}\cancel{8}} \\ -\ 55.19 \\ \hline 862.19 \end{array}$$

Sample 2

$$\begin{array}{r} 2.75 \\ 2.6\overline{)7.150} \\ 5\,2 \\ \hline 1\,95 \\ 1\,82 \\ \hline 130 \\ 130 \\ \hline 0 \end{array}$$

1. $\begin{array}{r} 49.92 \\ -\ 38.6 \\ \hline \end{array}$
2. $\begin{array}{r} 575.25 \\ -\ 9.009 \\ \hline \end{array}$
3. $\begin{array}{r} 337.14 \\ -\ 45.32 \\ \hline \end{array}$
4. $\begin{array}{r} 700.07 \\ -\ 38 \\ \hline \end{array}$
5. $\begin{array}{r} 3.4276 \\ -\ 0.828 \\ \hline \end{array}$
6. $\begin{array}{r} 16.8 \\ -\ 9.25 \\ \hline \end{array}$
7. $\begin{array}{r} 5.52 \\ -\ 4.763 \\ \hline \end{array}$
8. $\begin{array}{r} 877.3 \\ -\ 94.3 \\ \hline \end{array}$
9. $0.02\overline{)1.10}$
10. $0.8\overline{)0.036}$
11. $5.1\overline{)3376.2}$
12. $1.9\overline{)860.7}$
13. $3.4\overline{)0.0085}$
14. $0.05\overline{)2.367}$
15. $0.25\overline{)48}$
16. $0.34\overline{)1156}$

Express each fraction in simplest form.

Sample 3 $\frac{16}{24}$ **Solution** $\frac{16}{24} = \frac{2 \cdot 8}{3 \cdot 8} = \frac{2}{3}$

17. $\frac{14}{49}$
18. $\frac{21}{24}$
19. $\frac{39}{52}$
20. $\frac{27}{54}$
21. $\frac{63}{81}$
22. $\frac{27}{33}$

Perform the indicated operations. Express the answers in simplest form.

Sample 4 $\frac{5}{6} - \frac{2}{13}$ Note that the least common denominator is 78.

Solution $\frac{5}{6} - \frac{2}{13} = \left(\frac{5}{6} \cdot \frac{13}{13}\right) - \left(\frac{2}{13} \cdot \frac{6}{6}\right) = \frac{65}{78} - \frac{12}{78} = \frac{65 - 12}{78} = \frac{53}{78}$

Sample 5 $\frac{7}{18} \div \frac{14}{15}$ **Solution** $\frac{7}{18} \div \frac{14}{15} = \frac{7}{18} \cdot \frac{15}{14} = \frac{\overset{1}{\cancel{7}} \cdot \overset{5}{\cancel{15}}}{\underset{6}{\cancel{18}} \cdot \underset{2}{\cancel{14}}} = \frac{5}{12}$

23. $\frac{2}{3} - \frac{1}{6}$
24. $\frac{5}{7} - \frac{2}{3}$
25. $\frac{3}{4} - \frac{15}{21}$
26. $\frac{3}{4} - \frac{5}{12}$
27. $\frac{4}{5} - \frac{2}{11}$
28. $\frac{3}{7} - \frac{11}{28}$
29. $\frac{5}{6} \div \frac{5}{2}$
30. $\frac{12}{21} \div \frac{8}{14}$
31. $\frac{21}{25} \div \frac{9}{20}$
32. $\frac{10}{7} \div \frac{7}{10}$
33. $\frac{24}{25} \div \frac{5}{6}$
34. $\frac{24}{25} \div \frac{6}{5}$

Preparing for College Entrance Exams

Strategy for Success

Familiarize yourself with the test you will be taking well before the test date. This will help you to become comfortable with the types of questions and directions that may appear on the test. Sample tests, with explanations, are available for many standardized tests. The Preparing for College Entrance Exams tests at the end of even-numbered chapters in this book will give you helpful hints and practice in taking such tests.

Decide which is the best of the choices given and write the corresponding letter on your answer sheet.

1. Write an expression that corresponds to the following word sentence. The sum of two consecutive even integers is 20 less than their product.
(A) $n(n + 1) = n + (n + 1) - 20$ **(B)** $n + (n + 1) = n(n + 1) - 20$
(C) $n(n + 2) = n + (n + 2) - 20$ **(D)** $n + (n + 2) = n(n + 2) - 20$

2. Michael's weight is 85 lb less than twice the weight of his sister Stacy. Which equation represents the relationship between Michael's weight, m, and Stacy's weight, s?
(A) $s = 2m - 85$ **(B)** $s = 2m + 85$ **(C)** $m = 2s + 85$ **(D)** $m = 2s - 85$

3. Simplify the expression $73(19 + 31) - 48(24 + 26)$.
(A) 1250 **(B)** 6050 **(C)** 1200 **(D)** 1750

4. On a number line, point A has coordinate -4 and point B has coordinate 8. What is the coordinate of the point one fourth of the way from A to B?
(A) 2 **(B)** -2 **(C)** -1 **(D)** 5 **(E)** 6

5. The operation $*$ is defined for all real numbers a and b by $a * b = ab + a + b$. Which of the following properties does $*$ have?
I. Closure II. Commutativity
(A) I only **(B)** II only **(C)** I and II
(D) None of the above

6. Suppose x is a nonzero real number. Which of the following is (are) always true?
I. $\frac{1}{|x|} > 0$ II. $|x| > x$ III. $|x| > -x$
(A) I only **(B)** II only **(C)** III only
(D) II and III only **(E)** I and III only

7. a, b, c, and d are positive numbers. Which of the following guarantees that $\frac{a - b}{c - d} < 0$?
(A) $a > b$ and $c > d$ **(B)** $a < b$ and $c < d$ **(C)** $a > b$ and $c < d$
(D) $|a - b| > 0$ and $|c - d| > 0$ **(E)** $|a - b| > 0$ and $|c - d| < 0$