Math 112 - Quiz 16
November 16, 2018

Name Key

The following problems were selected from the practice problems, but some were slightly altered to simplify the algebra. Label answers appropriately. Show your work.

1. Differentiate $f(x) = e^x \cos x$. (2 pts.)

Using the product rule,

$$f'(x) = e^x \frac{d}{dx} \cos x + \cos x \frac{d}{dx} e^x = e^x(-\sin x) + (\cos x) e^x.$$

$$\boxed{\therefore \quad f'(x) = e^x(\cos x - \sin x)}$$

2. Differentiate $y = \dfrac{\sin t}{1+t}$. (2 pts.)

Using the Quotient rule,

$$y' = \frac{dy}{dt} = \frac{d}{dt}\left(\frac{\sin t}{1+t}\right)$$

$$= \frac{(1+t)\frac{d}{dt}\sin t - \sin t \frac{d}{dt}(1+t)}{(1+t)^2} = \frac{(1+t)\cos t - (\sin t)(1)}{(1+t)^2}.$$

$$\therefore \quad y' = \frac{(1+t)\cos t - \sin t}{(1+t)^2} = \frac{\cos t}{1+t} - \frac{\sin t}{(1+t)^2}.$$

3. Differentiate $y = \tan \pi x$. (2 pts.)

By the Chain rule and using that $\frac{d}{dt}\tan t = \sec^2 t$,

we have

$$\frac{dy}{dx} = \frac{d}{dx}\tan \pi x = \sec^2 \pi x \cdot \frac{d}{dx}(\pi x) = \pi \sec^2 \pi x.$$

That is, $\boxed{\frac{d}{dx}\tan \pi x = \pi \sec^2 \pi x}$

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4. What special limit that was derived in class (and in the textbook) can be used to find

$$\lim_{x\to 0} \frac{\sin 2x}{3x}?$$

Now use it to find the above limit. (2 pts.)

The limit is $\lim_{t\to 0} \frac{\sin t}{t} = 1$.

$$\lim_{x\to 0} \frac{\sin 2x}{3x} = \lim_{x\to 0} \left(\frac{\sin 2x}{2x} \cdot \frac{2x}{3x}\right) = \lim_{x\to 0} \left(\frac{2}{3} \cdot \frac{\sin 2x}{2x}\right)$$

$$= \frac{2}{3} \lim_{x\to 0} \frac{\sin 2x}{2x}$$

$$= \frac{2}{3}(1) = \boxed{2/3}$$

5. A mass vibrates horizontally on a smooth horizontal table. Its equation of motion is $x(t) = 8 \sin t$, where t is in seconds and x in centimeters. Find its acceleration at time $t = \pi/3$ seconds. (2 pts.)

The formula for velocity is

$$v(t) = \frac{d}{dt}(8 \sin t) = 8 \cos t.$$

The formula for acceleration is

$$a(t) = \frac{d}{dt} v(t) = \frac{d}{dt}(8 \cos t) = -8 \sin t$$

Hence, at $t = \frac{\pi}{3}$, we have

$$a(\pi/3) = -8 \sin\left(\frac{\pi}{3}\right) = -8\left(\frac{\sqrt{3}}{2}\right) = -4\sqrt{3}\ \frac{cm}{s^2}$$

Extra credit. (2 pts.) Find the derivative of $y = \sqrt{5x+1}$.

This is the composite fnct $y = f(g(x))$ where $f(x) = \sqrt{x}$ and $g(x) = 5x+1$. By the Chain Rule,

$$\frac{dy}{dx} = f'(g(x))\, g'(x) = \frac{1}{2\sqrt{g(x)}} \cdot \frac{d}{dx}(5x+1)$$

$$= \frac{1}{2\sqrt{5x+1}} \cdot 5$$

$$\therefore \frac{d}{dx}\sqrt{5x+1} = \frac{5}{2\sqrt{5x+1}}$$