

Memorize the derivative formulas given below in Table 1 and the differentiation rules that follow it.

TABLE 1. **Basic Derivative Formulas**

$f(x)$	$f'(x)$
c (c , a constant)	0
x	1
x^n (n , any real number)	nx^{n-1}
e^x	e^x
b^x	$b^x \ln b$
$\ln x $	$\frac{1}{x}$
$\log_b x $	$\frac{1}{x \ln b}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$

Let f and g denote functions that are differentiable on an interval (a, b) . Let c denote a constant. For x in (a, b) , there are the following differentiation rules.

Constant Multiple Rule

$$\frac{d}{dx}[cf(x)] = cf'(x)$$

In words: *The derivative of a constant times a function is equal to the constant times the derivative of the function.*

Sum & Difference Rules

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

In words: *The derivative of a sum (difference) is equal to the sum (difference) of the derivatives.*

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

In words: *The derivative of a product of two functions is equal to the first function times the derivative of the second function plus the second function times the derivative of the first function.*

Quotient Rule

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \quad (\text{provided } g(x) \neq 0)$$

Mnemonic: *Low d'high minus high d'low over the square of what's below.*

Chain Rule

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

In words: *The derivative of a composite function is equal to the derivative of the outer function evaluated at the inner function times the derivative of the inner function.*

Chain Rule using Leibniz notation

If $y = f(g(x))$ is rewritten in the form $y = f(u)$, where $u = g(x)$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

Power Rule combined with the Chain Rule

Let n be a real number. Let $u = g(x)$ be a differentiable function. Then

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} \frac{d}{dx}g(x) \quad \text{or} \quad \frac{d}{dx}u^n = \frac{d}{du}u^n \cdot \frac{du}{dx} = nu^{n-1} \frac{du}{dx}.$$

Other differentiation formulas combined with the Chain Rule

If the variable x of a function listed in Table 1 is replaced with a differentiable function $u = g(x)$, then the chain rule must be incorporated into the formula, such as above for the power function. Other examples are:

$$\frac{d}{dx}e^u = e^u \frac{du}{dx}, \quad \frac{d}{dx} \ln |u| = \frac{1}{u} \frac{du}{dx}, \quad \frac{d}{dx} \sin u = \cos u \frac{du}{dx}, \quad \frac{d}{dx} \cos u = -\sin u \frac{du}{dx}, \quad \frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx},$$

$$\frac{d}{dx} \sec u = \sec u \tan u \frac{du}{dx}, \text{ and so on.}$$