

3 Solving Equations and Problems

3-1 Transforming Equations: Addition and Subtraction

Objective: To solve equations using addition and subtraction.

Properties

Addition Property of Equality If the same number is added to equal numbers, the sums are equal.

Subtraction Property of Equality If the same number is subtracted from equal numbers, the differences are equal.

Vocabulary

Equivalent equations Equations that have the same solution set over a given domain.

Transformations Operations on an equation that produce a simpler equivalent equation.

By substitution You can substitute an equivalent expression for any expression in an equation. You do this when you simplify an expression in an equation.

By addition You can add the same number to each side of an equation.

By subtraction You can subtract the same number from each side of an equation.

CAUTION

To check your work, you should check that each solution of the final equation satisfies the *original equation*.

Example 1 Solve $x - 6 = 11$.

Solution

$$\begin{array}{l} x - 6 = 11 \\ x - 6 + 6 = 11 + 6 \\ x = 17 \end{array} \quad \left\{ \begin{array}{l} \text{To get } x \text{ alone on one side,} \\ \text{add 6 to each side and then simplify.} \end{array} \right.$$

Check: $x - 6 = 11$ ← Original equation.

$$\begin{array}{l} 17 - 6 \stackrel{?}{=} 11 \\ 11 = 11 \checkmark \end{array} \quad \begin{array}{l} \text{Substitute 17 for } x. \end{array}$$

The solution set is $\{17\}$.

Solve.

1. $a - 9 = 11$

2. $b - 5 = 13$

3. $x - 20 = -19$

4. $d - 14 = 5$

5. $x - 15 = 0$

6. $v - 27 = -54$

7. $x - 6 = 27$

8. $q - 7 = 11$

9. $q - 9 = -16$

3-1 Transforming Equations: Addition and Subtraction (continued)

Example 2 Solve $-9 = n + 11$.

Solution

$$\begin{array}{l} -9 = n + 11 \\ -9 - 11 = n + 11 - 11 \\ -20 = n \end{array} \quad \left\{ \begin{array}{l} \text{To get } n \text{ alone on one side,} \\ \text{subtract 11 from each side.} \\ \text{Simplify.} \end{array} \right.$$

Check: $-9 = n + 11$ ← Original equation

$$\begin{array}{l} -9 \stackrel{?}{=} -20 + 11 \\ -9 = -9 \checkmark \end{array} \quad \begin{array}{l} \text{Substitute } -20 \text{ for } n. \\ \text{The solution set is } \{-20\}. \end{array}$$

Solve.

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|---------------------|--------------------|---------------------|---------------------|
| 10. $-6 = m + 6$ | 11. $21 = x + 15$ | 12. $-26 + m = 24$ | 13. $-37 + n = 63$ |
| 14. $p + 18 = -22$ | 15. $a + 60 = -15$ | 16. $14 + t = 0$ | 17. $29 = y - 12$ |
| 18. $35 = x + 16$ | 19. $-4 = u - 6$ | 20. $22 = y + 3$ | 21. $c - 8 = -10$ |
| 22. $x + 1.5 = 6.8$ | 23. $-1 + a = 0.5$ | 24. $3.9 = y - 1.4$ | 25. $7.5 = w - 2.5$ |

Example 3 Solve $-x + 5 = 4$.

Solution

$$\begin{array}{l} -x + 5 = 4 \\ -x + 5 - 5 = 4 - 5 \\ -x = -1 \\ x = 1 \end{array} \quad \left\{ \begin{array}{l} \text{To get } x \text{ alone on one side,} \\ \text{subtract 5 from each side and simplify.} \\ \text{If the opposite of a number is } -1, \\ \text{the number must be 1.} \end{array} \right.$$

Check: $-x + 5 = 4$ ← Original equation

$$\begin{array}{l} -1 + 5 \stackrel{?}{=} 4 \\ 4 = 4 \checkmark \end{array} \quad \begin{array}{l} \text{Substitute 1 for } x. \\ \text{The solution set is } \{1\}. \end{array}$$

Solve.

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|------------------|--------------------|--------------------|
| 26. $-x + 3 = 5$ | 27. $-y + 7 = 17$ | 28. $12 - x = 18$ |
| 29. $7 - y = 11$ | 30. $9 = -x + 16$ | 31. $13 = 22 - y$ |
| 32. $-5 - y = 7$ | 33. $10 = -12 - e$ | 34. $15 = -y + 10$ |

Mixed Review Exercises

Evaluate if $a = 3$, $b = -6$, $c = -4$, and $d = 2$.

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|---------------------------|----------------------------|------------------------|
| 1. $a - b - c $ | 2. $(c - d) - (b - a)$ | 3. $3 c - (-b)$ |
| 4. $\frac{a - 2b}{a + d}$ | 5. $\frac{3b + c - d}{ad}$ | 6. $\frac{2ab}{c + d}$ |

Simplify.

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|------------------|------------------------------------|---------------------|
| 7. $(-3)(-4)(8)$ | 8. $(-7 \cdot 16) + (-7 \cdot 24)$ | 9. $252 \div (-36)$ |
|------------------|------------------------------------|---------------------|