Algebra

absolute value $\lvert x-y \rvert$	the distance between 2 points, x and y, on a number line
absolute value $\lvert x \rvert$	$\lvert x \rvert = \begin{cases} x, & if\ x \ge 0 \\ -x, & if\ x < 0 \end{cases}$ $\lvert x \rvert$ expresses the distance from x to zero on the number line
distance formula	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ = distance between 2 points (x_1, y_1) and (x_2, y_2) on coordinate plane
midpoint formula	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ = (average of the x coordinates, average the y coordinates)
slope of a line between any 2 points (x_1, y_1) and (x_2, y_2)	$m = \frac{y_2 - y_1}{x_2 - x_1}$ slope is the measure of the steepness of a line **horizontal line** slope = 0; vertical line slope is undefined **parallel lines** have equal slopes; **perpendicular lines** - slopes are opposite (negative) reciprocals;product of slopes = -1
linear equation slope intercept form point-slope form standard form	 y = mx + b, where m = slope and b = y coordinate of the y-intercept $y - y_1 = m(x - x_1)$ Ax + By = C
equations for a circle	$x^2 + y^2 = r^2$ circle centered at (0, 0) with radius r $(x-h)^2 + (y-k)^2 = r^2$ circle centered at (h, k) with radius r
equations for parabola	$y = a(x-h)^2 + k$ parabola opens up or down; vertex at (h, k) $x = a(y-k)^2 + h$ parabola opens left or right; vertex at (h, k)
factoring difference of 2 squares sum of 2 squares difference of 2 cubes sum of 2 cubes perfect square trinomials	 $a^2 - b^2 = (a+b)(a-b)$ prime (not factorable) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ $(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$ 1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1
quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ yields solutions for $ax^2 + bx + c = 0$ Note: the sum of the two solutions is $-b/a$ and product of solutions = c/a
discriminant test	$b^2 - 4ac = 0$ ➔ one real solution $b^2 - 4ac > 0$ ➔ two real solutions $b^2 - 4ac < 0$ ➔ two complex conjugate solutions
exponent rules	$x^0 = 1$ $\left(x^a\right)^b = x^{ab}$ $x^a x^b = x^{a+b}$ $\frac{x^a}{x^b} = x^{a-b}$ $x^{-a} = \frac{1}{x^a}$ and $\frac{1}{x^{-a}} = x^a$
radicals and exponents	$\sqrt{x} = x^{1/2}$ $\sqrt[3]{x} = x^{1/3}$ $\sqrt[b]{x^a} = x^{a/b}$
complex number	$i = \sqrt{-1}$ $i^2 = -1$ $i^3 = -i$ $i^4 = 1$
absolute value equation	If |**algebraic expression**| = A, then set up two equations and solve: **algebraic expression** = A and **algebraic expression** = -A
absolute value inequalities	If |algebraic expression| < A, then set up compound inequality (conjunction) and solve: **-A < algebraic expression < A** If |algebraic expression| > A, then set up two inequalities (disjunction) and solve: **algebraic expression > A *or* algebraic expression < -A**

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Algebra

distance	$d = rt$ distance = rate x time
average (mean)	$average = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$
% change	$percent\ change = \frac{change\ in\ quantity}{original\ quantity}$
proportion	$\frac{a}{b} = \frac{c}{d}$ $implies\ ad = bc$ by cross-multiplication
direct variation	$y = kx$ or $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ where k is the constant of variation
inverse variation	$xy = k$ or $x_1 y_1 = x_2 y_2$ where k is the constant of variation
arithmetic sequence	$a_n = a_1 + (n-1)d$, nth term of arithmetic sequence, d is the common difference between terms
arithmetic series	$S_n = \frac{n(a_1 + a_n)}{2}$, S_n = sum of first "n" terms of a finite geometric series; a_1 = first term
geometric sequence	$a_n = a_1 r^{n-1}$ nth term of the geometric series; r is the common ratio
geometric series	$S_n = a_1 + a_1 r + \ldots + a_1 r^{n-1} = \frac{a_1(1-r^n)}{1-r}$, S_n = sum of first "n" terms of geometric series
probability	Probability of an outcome happening = $\frac{\text{number of desired outcomes}}{\text{total number of possible outcomes}}$ Probability of two mutually exclusive events, A and B, happening = P(A) • P(B)
combinations	A combination means the order of the elements doesn't matter. For example, a shirt and pants is the same thing as pants and a shirt. Possible combinations of 3 shirts and 4 pants = 3 x 4 = 12. ${}_nC_r = \frac{n!}{r!(n-r)!}$ number of combinations of n items taken r at a time; order does not matter;
permutations	${}_nP_r = \frac{n!}{(n-r)!}$ number of ways to arrange n items taken r at a time; order does matter;
logarithms	$\log_b y = x$ $means$ $y = b^x$ $\log_b MN = \log_b M + \log_b N$ $\log_b \left(M/N\right) = \log_b M - \log_b N$ $\log_b M^N = N \log_b M$ $\log_b M = \frac{\log_c M}{\log_c b}$ (change of base formula)
Transformations Translation (no rotation or size change)	(x, y) → (x + a, y + b) represents horizontal shift of "a" units, vertical shift of "b" units);
Reflection (flip) Over x-axis Over y-axis Over line y = x Over origin (line y= -x)	 (x, y) → (x, -y) (x, y) → (-x, y) (x, y) → (y, x) (x, y) → (-y, -x)
Rotation about origin 90° CCW or 270° CW 180° CCW or 180° CW 270° CCW or 90° CW	 (x, y) → (-y, x) (x, y) → (-x, -y) (x, y) → (y, -x)

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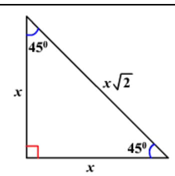
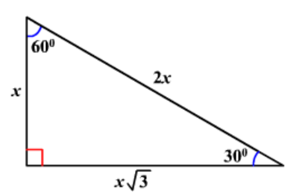
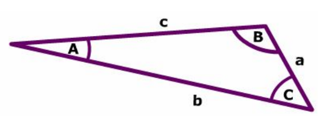
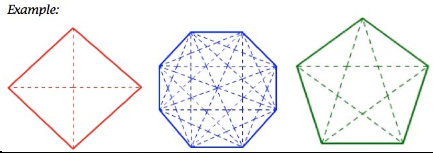
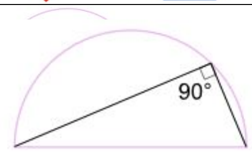
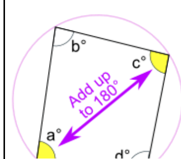
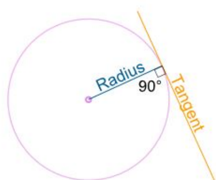
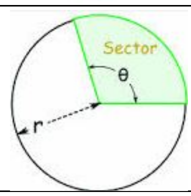
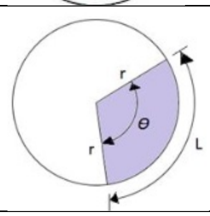
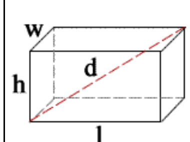
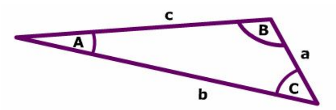
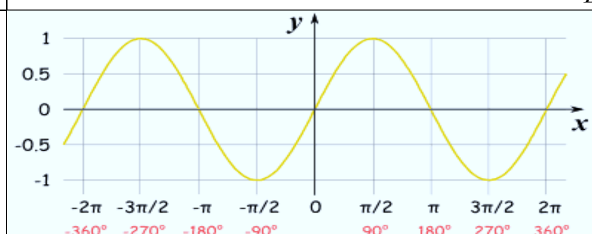
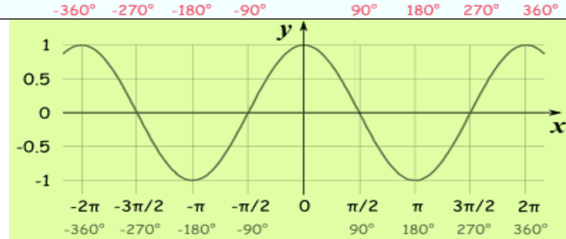
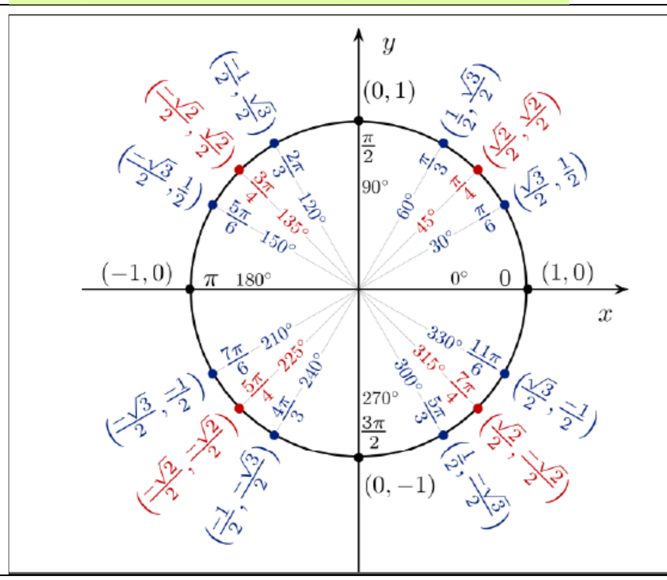
Geometry

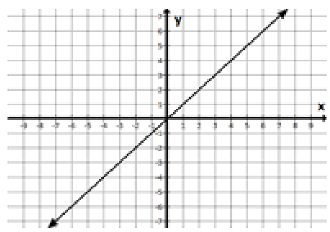
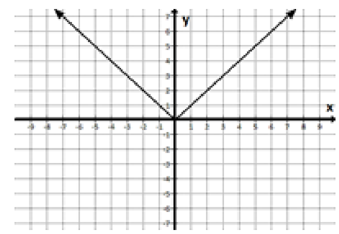
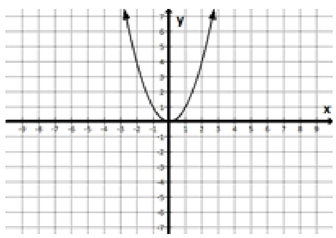
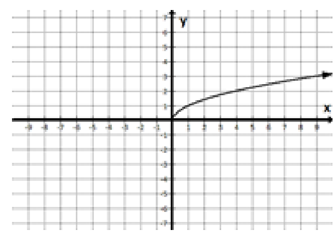
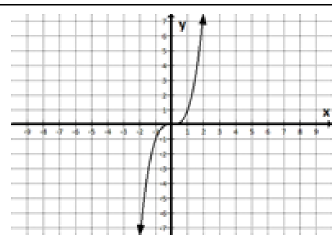
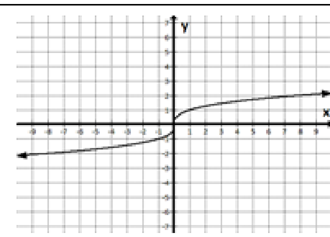
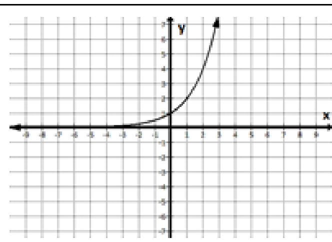
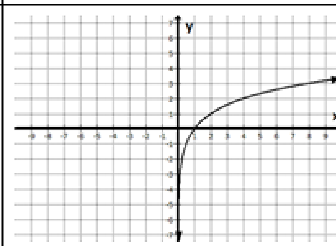
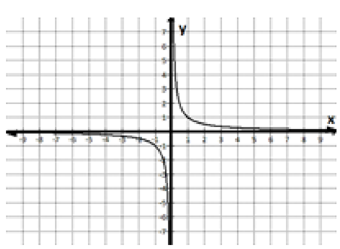
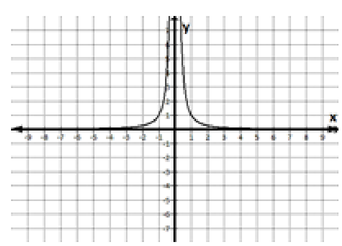
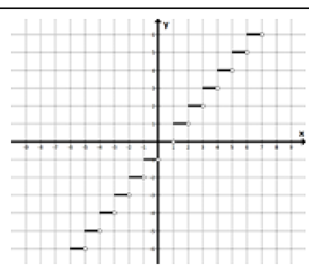
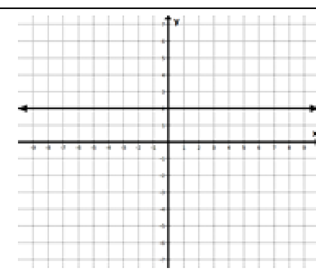
Perimeter	**In general, perimeter = sum of lengths of sides**
square	P = 4s
rectangle	P = 2L + 2W
circle	C = $2\pi r$ (circumference)
Area	**Note that the units are square units.**
square	$A = s^2$
rectangle, parallelogram	A = bh (note base and height are ***always*** perpendicular)
triangle	A = ½ bh
kite	A = ½ (d_1 d_2) where d_1 and d_2 are lengths of the diagonals
circle	$A = \pi r^2$
trapezoid	A = ½ ($b_1 + b_2$)h (average of bases times the height)
Volume	**B = area of base; h = height**
cube	$V = Bh = s^3$ where s is the side length
rectangular prism	$V = Bh = lwh$
sphere	$V = \frac{4}{3}\pi r^3$
cone	$V = \frac{1}{3}\pi r^2 h$
cylinder	$V = Bh = \pi r^2 h$
Triangles	
Congruency Theorems	SSS, SAS, ASA, or AAS or use H-L (right triangles only)
Pythagorean Theorem	$c^2 = a^2 + b^2$ is used to find length of sides or hypotenuse, c, for a right triangle If $c^2 = a^2 + b^2$, then the triangle is a right triangle (Converse of Pythagorean Theorem) If $c^2 < a^2 + b^2$, then the triangle is acute If $c^2 > a^2 + b^2$, then the triangle is obtuse
Common Pythagorean Triples	3 – 4 – 5 5 – 12 – 13 7 – 24 –25 8 – 15 – 17 6 – 8 – 10 10 – 24 – 26 14 – 48 – 50 16 – 30 – 34
45-45-90 triangle	 ratios of sides lengths: $1x : 1x : x\sqrt{2}$
30-60-90 triangle	 ratios of sides lengths: $1x : x\sqrt{3} : 2x$
Law of Sines and Law of Cosines (for non-right triangles)	For **non**-right triangles, use the Law of Sines to find side lengths and angles when possible, or use Law of Cosines when you have 2 sides and the included angle .  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ $c^2 = a^2 + b^2 - 2bc \cos C$

Other Formulas and Facts	
sum of interior angles of a convex polygon	S = (n – 2)180° , n = # of sides
sum of exterior angles in a convex polygon	sum of exterior angles always equals 360°
number of diagonals in a convex polygon	*Example:*  $\#\text{ of diagnonals} = \frac{n(n-3)}{2}$, where n = number of sides
inscribed angle facts	 An inscribed angle a is half the central angle, 2a. Therefore, the inscribed angle 90° is half of the central angle 180°.  A Cyclic Quadrilateral's opposite angles add up to 180°: a + c = 180° b + d = 180°  A tangent is a line that just touches a circle at one point. It always forms a right angle with the circle's radius to the point of tangency.
sector area of a circle	 $A = \frac{\theta^o}{360^o} \bullet \pi r^2$ = fractional part of the circle's area
length of intercepted arc	 $L = \frac{\theta^o}{360^o} \bullet 2\pi r$ = fractional part of the circumference
Length of diagonal of a rectangular prism	 $d = \sqrt{L^2 + H^2 + W^2}$
Vectors	<a, b> + <c, d> = <a +c, b + d> similar to add complex numbers (a +bi) + (c + di) = (a+b) + i(c + d)

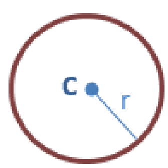
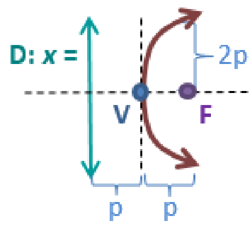
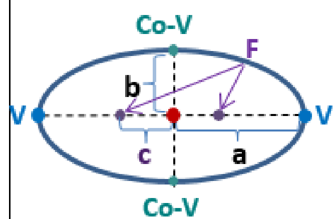
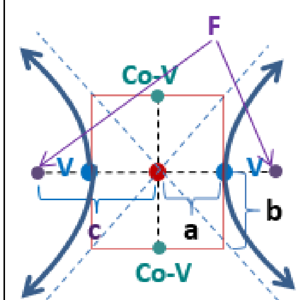
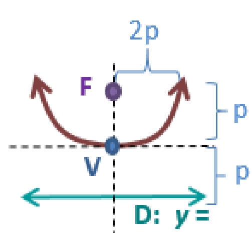
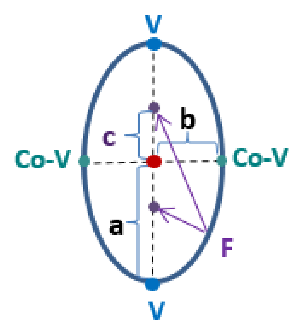
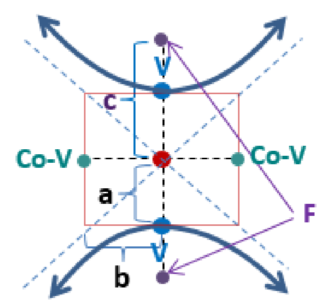
Trigonometry

Trigonometric ratios	$\sin\Theta = \frac{opp}{hyp} = \frac{y}{r}$ $\csc\Theta = \frac{1}{\sin\Theta} = \frac{hyp}{opp} = \frac{r}{y}$ $\cos\Theta = \frac{adj}{hyp} = \frac{x}{r}$ $\sec\Theta = \frac{1}{\cos\Theta} = \frac{hyp}{adj} = \frac{r}{x}$ $\tan\Theta = \frac{\sin\Theta}{\cos\Theta} = \frac{opp}{adj} = \frac{y}{x}$ $\cot\Theta = \frac{\cos\Theta}{\sin\Theta} = \frac{adj}{opp} = \frac{x}{y}$
Trig identities	$\sin^2\Theta + \cos^2\Theta = 1$ $1 + \cot^2\Theta = \csc^2\Theta$ $1 + \tan^2\Theta = \sec^2\Theta$
Law of Sines and Law of Cosines (for non-right triangles)	For **non**-right triangles, use the Law of Sines to find side lengths and angles when possible, or use Law of Cosines when you have 2 sides and the included angle .  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ $c^2 = a^2 + b^2 - 2bc\cos C$
Sine & Cosine functions	$y = A\sin(Bx + C) + D$ $y = A\cos(Bx + C) + D$ $Amplitude = \|A\|$ $Period = \frac{2\pi}{B}$ $Horizontal\ shift = -\frac{C}{B}$ $Vertical\ shift = D$
Graph of $y = \sin x$	
Graph of $y = \cos x$	
Unit Circle $x = \cos\theta$ $y = \sin\theta$ 180° = π radians	 **degrees** — **radians** 0 — 0 30 — $\pi/6$ 45 — $\pi/4$ 60 — $\pi/3$ 90 — $\pi/2$ 180 — π 270 — $3\pi/2$ 360 — 2π

Graphs

Parent Function	Graph	Parent Function	Graph
$y=x$ **Linear**, Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$ End Behavior: $x\to-\infty,\ y\to-\infty$ $x\to\infty,\ y\to\infty$		$y=\lvert x\rvert$ **Absolute Value**, Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$ End Behavior: $x\to-\infty,\ y\to\infty$ $x\to\infty,\ y\to\infty$	
$y=x^2$ **Quadratic**, Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$ End Behavior: $x\to-\infty,\ y\to\infty$ $x\to\infty,\ y\to\infty$		$y=\sqrt{x}$ **Radical**, Neither Domain: $[0,\infty)$ Range: $[0,\infty)$ End Behavior: $x\to\infty,\ y\to\infty$	
$y=x^3$ **Cubic**, Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$ End Behavior: $x\to-\infty,\ y\to-\infty$ $x\to\infty,\ y\to\infty$		$y=\sqrt[3]{x}$ **Cube Root**, Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$ End Behavior: $x\to-\infty,\ y\to-\infty$ $x\to\infty,\ y\to\infty$	
$y=b^x,\ b>1$ **Exponential**, Neither Domain: $(-\infty,\infty)$ Range: $(0,\infty)$ End Behavior: $x\to-\infty,\ y\to0$ $x\to\infty,\ y\to\infty$		$y=\log_b(x),\ b>1$ **Log**, Neither Domain: $(0,\infty)$ Range: $(-\infty,\infty)$ End Behavior: $x\to0^+,\ y\to-\infty$ $x\to\infty,\ y\to\infty$	
$y=\frac{1}{x}$ **Rational (Inverse)**, Odd Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(-\infty,0)\cup(0,\infty)$ End Behavior: $x\to-\infty,\ y\to0$ $x\to\infty,\ y\to0$		$y=\frac{1}{x^2}$ **Rational (Inverse Squared)**, Even Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(0,\infty)$ End Behavior: $x\to-\infty,\ y\to0$ $x\to\infty,\ y\to0$	
$y=\text{int}(x)=[x]$ **Greatest Integer**, Neither Domain: $(-\infty,\infty)$ Range: $\{y: y\in\mathbb{Z}\}$ (integers) End Behavior: $x\to-\infty,\ y\to-\infty$ $x\to\infty,\ y\to\infty$		$y=C$ ($y=2$ in the graph) **Constant**, Even Domain: $(-\infty,\infty)$ Range: $\{y: y=C\}$ End Behavior: $x\to-\infty,\ y\to C$ $x\to\infty,\ y\to C$	

Conic Sections

Conic: / Equation:	**Circle** Center: (h, k)	**Parabola** Vertex: (h, k)	**Ellipse** Center: (h, k) a always larger than b	**Hyperbola** Center: (h, k) a always before the "—"
Equation (Horizontal)	$(x-h)^2+(y-k)^2=r^2$	$x=a(y-k)^2+h$ or $x-h=a(y-k)^2$ or $b(x-h)=(y-k)^2$	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ Asymptotes: $y-k=\pm\frac{b}{a}(x-h)$
Graph (Horizontal)		**(Positive Coefficient)** 		
Equation (Vertical)	Same	$y=a(x-h)^2+k$ or $y-k=a(x-h)^2$ or $b(y-k)=(x-h)^2$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$ Asymptotes: $y-k=\pm\frac{a}{b}(x-h)$
Graph (Vertical)	Same	**(Positive Coefficient)** 		
Additional Information	To get ***r***: $y=\pm\sqrt{r^2-(x-h)^2}+k$	For $x-h=a(y-k)^2$: $p=\frac{1}{4a}$; $a=\frac{1}{4p}$ For $b(x-h)=(y-k)^2$: $b=4p$; $p=\frac{b}{4}$ p = **focal length** **Negative Coefficients:** Flip parabola	$c^2=a^2-b^2$	$c^2=a^2+b^2$

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