Differentiation Rules and Formulas

Let f and g be functions that are differentiable on an interval I. For $x \in I$, the following hold:

Constant Multiple Rule

$$\frac{d}{dx}[cf(x)] = cf'(x) \qquad \text{(any constant } c\text{)}$$

The derivative of a constant times a function is equal to the constant times the derivative of the function.

Sum & Difference Rules

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

The derivative of a sum (difference) is equal to the sum (difference) of the derivatives.

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

The derivative of a product of two functions is equal to the first one times the derivative of the second one plus the second one times the derivative of the first one.

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \qquad \text{(provided } g(x) \neq 0\text{)}$$

Mnemonic: *Low d'high minus high d'low over the square of what's below.*

Chain Rule (prime notation)

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

The derivative of a composite function is equal to the derivative of the outer function evaluated at the inner function times the derivative of the inner function.

Chain Rule (Leibniz notation)

Let $y = f(u)$ and $u = g(x)$.

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Generalized Power Rule (Power Rule combined with the Chain Rule)

Let n be a real number. Let $u = g(x)$ be a differentiable function.

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}\frac{d}{dx}g(x) \quad \text{or} \quad \frac{d}{dx}u^n = \frac{d}{du}u^n \cdot \frac{du}{dx} = nu^{n-1}\frac{du}{dx}$$

Similarly, for the functions listed in Table 1,

$$\frac{d}{dx}e^u = e^u\frac{du}{dx}, \qquad \frac{d}{dx}\ln|u| = \frac{1}{u}\frac{du}{dx}, \qquad \frac{d}{dx}\sin u = \cos u\frac{du}{dx}, \qquad \frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx},$$

and so on.

TABLE 1. **Basic Derivatives**

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
c	0	$*$	$*$
x	1	x^n	nx^{n-1}
e^x	e^x	b^x	$b^x \ln b$
$\ln \lvert x\rvert$	$\frac{1}{x}$	$\log_b \lvert x\rvert$	$\frac{1}{x \ln b}$
$\sin x$	$\cos x$	$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$	$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$	$\csc x$	$-\csc x \cot x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$	$\cot^{-1} x$	$-\frac{1}{1+x^2}$
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$	$\csc^{-1} x$	$-\frac{1}{x\sqrt{x^2-1}}$

In the above table, b, c, and n denote constants, where $b > 0$.

Note. Unfortunately, the two differentiation formulas in the last row are not universally accepted. The reason for this is that there is no general agreement on the definition of the inverse secant and the inverse cosecant functions. These formulas are used if the inverse secant function is defined by

$$y = \sec^{-1} x \ (|x| \geq 1) \quad \text{if and only if} \quad \sec y = x \text{ and } y \in [0, \tfrac{\pi}{2}) \cup [\pi, \tfrac{3\pi}{2})$$

and the inverse secant function by

$$y = \csc^{-1} x \ (|x| \geq 1) \quad \text{if and only if} \quad \csc y = x \text{ and } y \in (0, \tfrac{\pi}{2}] \cup (\pi, \tfrac{3\pi}{2}].$$

However, other authors define $y = \sec^{-1} x$ by choosing y from the set $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$. And for $y = \csc^{-1} x$, they choose from $[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$. In that case, the corresponding differentiation formulas are

$$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}} \quad \text{and} \quad \frac{d}{dx} \csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}.$$