

Section 5.3

#43, p. 400. Evaluate $\int_0^{\pi} f(x) dx$ where the function f is the following piecewise-defined function:

$$f := x \rightarrow \begin{cases} \sin(x) & 0 \leq x < \frac{\pi}{2} \\ \cos(x) & \frac{\pi}{2} \leq x \leq \pi \end{cases}$$

$$f := x \rightarrow \text{piecewise}\left(0 \leq x \text{ and } x < \frac{1}{2} \pi, \sin(x), \frac{1}{2} \pi \leq x \text{ and } x \leq \pi, \cos(x)\right) \quad (1)$$

$f(x)$

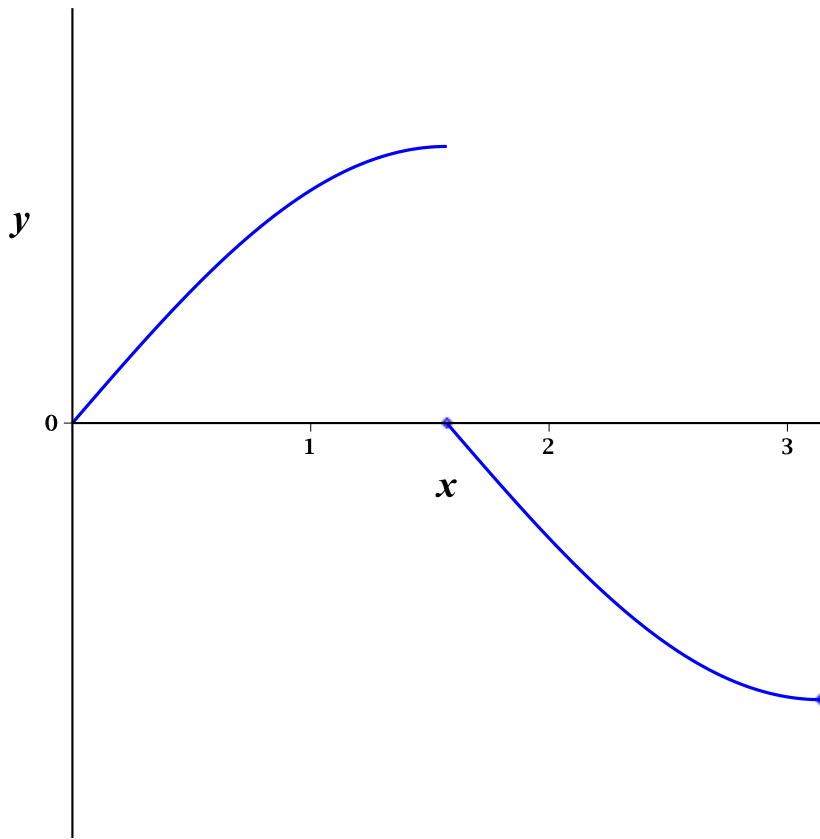
$$\begin{cases} \sin(x) & 0 \leq x \text{ and } x < \frac{1}{2} \pi \\ \cos(x) & \frac{1}{2} \pi \leq x \text{ and } x \leq \pi \end{cases} \quad (2)$$

$$f(0), f\left(\frac{\pi}{3}\right), f\left(\frac{\pi}{2}\right), f\left(\frac{2\pi}{3}\right), f(\pi)$$

$$0, \frac{1}{2} \sqrt{3}, 0, -\frac{1}{2}, -1 \quad (3)$$

Solution.

> `plot(f(x), x=0..pi, y=-1.5..1.5, tickmarks = ([spacing(1), spacing(2)]), thickness=1, font = [Times, bold, 8], labelfont = [Roman, bold, 14], color = blue, discount = true)`



> $\int_0^{\frac{\pi}{2}} f(x) dx$

1

(4)

> $\int_{\frac{\pi}{2}}^{\pi} f(x) dx$

-1

(5)

```
> \int_0^\pi f(x) dx
```

0

(6)

```
Area:
```

```
> \int_0^\pi |f(x)| dx
```

$$\int_0^\pi \begin{cases} \sin(x) & 0 \leq x \text{ and } x < \frac{1}{2} \pi \\ \cos(x) & \frac{1}{2} \pi \leq x \text{ and } x \leq \pi \end{cases} dx$$

(7)

```
> evalf(%, 3)
```

2.00

(8)

```
>
```