Math 122 - Quiz 2
January 18, 2019

Name Key

No calculators. Show all necessary work.

1. Given the relation $9x^2 - y^2 = 1$. Find dy/dx by first solving the equation explicitly for y and then differentiating. (3 points.)

$9x^2 - y^2 = 1 \Rightarrow y^2 = 9x^2 - 1 \Rightarrow y = \pm\sqrt{9x^2-1}$

Differentiating, we have

$y = \pm \frac{d}{dx}(9x^2-1)^{1/2} = \pm\frac{1}{2}(9x^2-1)^{-1/2}\frac{d}{dx}(9x^2-1)$

$= \pm\frac{1}{2}\left(\frac{18x}{\sqrt{9x^2-1}}\right) = \pm\frac{9x}{\sqrt{9x^2-1}}$

2. If $f(x) + x^2[f(x)]^3 = 10$ and $f(1) = 2$, find $f'(1)$. (3 pts.)

Differentiating wrt x, we get $\frac{d}{dx}f(x) + \frac{d}{dx}x^2[f(x)]^3 = \frac{d}{dx}10$

$\Rightarrow f'(x) + x^2\left(3[f(x)]^2 f'(x)\right) + 2x[f(x)]^3 = 0$

Let $x=1$. Then we have

$f'(1) + 3[f(1)]^2 f'(1) + 2[f(1)]^3 = 0 \Rightarrow f'(1) + 3(2^2)f'(1) + 2(2^3) = 0$

$\Rightarrow 13f'(1) = -16 \Rightarrow f'(1) = -\frac{16}{13}$

3. Find an equation of the tangent line to the curve $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$ at $(0, 1/2)$. (4 pts.)

Diff. wrt x, we have: $2x + 2yy' = 2(2x^2+2y^2-x)\frac{d}{dx}(2x^2+2y^2-x)$

$\Rightarrow 2x + 2yy' = (4x^2+4y^2-2x)(4x+4yy'-1)$

Now let $x=0$ & $y=\frac{1}{2}$:

$2(\tfrac{1}{2})y' = \left(4(\tfrac{1}{2})^2\right)\left(4(\tfrac{1}{2})y'-1\right)$

$\Rightarrow y' = 2y' - 1 \Rightarrow \underline{y' = 1}$

The eqn of the line with slope $m=1$ containing the point $(0, \frac{1}{2})$ is

$y - \frac{1}{2} = 1(x-0) \Rightarrow y = x + \frac{1}{2}$

Extra Credit. (1 pt.)
In Problem 1, find dy/dx by implicit differentiation.

$\frac{d}{dx}(9x^2-y^2) = \frac{d}{dx}1 \Rightarrow 18x - 2yy' = 0 \Rightarrow 2yy' = 18x$

$\Rightarrow y' = \frac{9x}{y}$