Samantha Wood
Math 122
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Quiz 14

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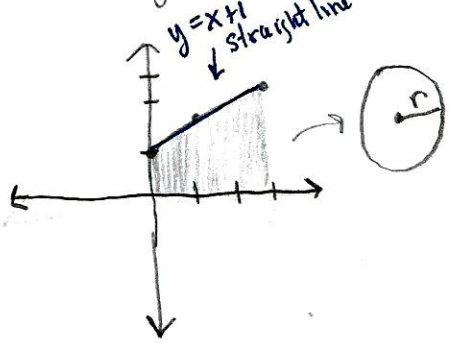
1.) Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, solid, and a typical disk.

$y = x+1$, $y=0$, $x=0$, $x=2$; about the x-axis.



x	$y=x+1$
0	1
1	2
2	3

$A(x) = \pi y^2 = \pi(x+1)^2$ (Cross section)

$$V = \int_0^2 A(x)\,dx = \pi \int_0^2 (x+1)^2\,dx$$

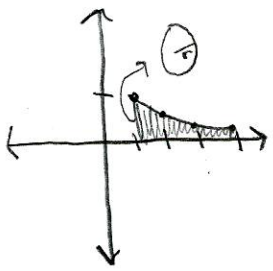
$$= \pi \left[\frac{(x+1)^3}{3}\right]_0^2$$

$$= \pi \left[\frac{(2+1)^3}{3} - \frac{(0+1)^3}{3}\right]$$

$$V = \int_0^2 A(x)\,dx = \pi\left[\frac{27}{3} - \frac{8}{3}\right] = \boxed{\frac{26}{3}\pi}$$ ✓

2.) Find the volume of the solid obtained by rotating the region by the given curves about the specified line. Sketch the region, solid, and a typical disk.

$y = \frac{1}{x}$, $y=0$, $x=1$, $x=4$; about the x-axis.



x	$y = \frac{1}{x}$
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$
4	$\frac{1}{4}$

$A(x) = \pi y^2 = \pi\left(\frac{1}{x}\right)^2$

$$V = \int_1^4 A(x)\,dx = \pi \int_1^4 \left(\frac{1}{x}\right)^2 dx$$

$$= \pi\left[-\frac{1}{x}\right]_1^4$$ ✓

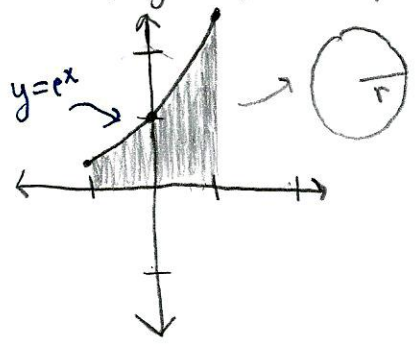
$$= \pi\left[-\frac{1}{4} - \left(-\frac{1}{1}\right)\right]$$

$$= \pi\left(-\frac{1}{4} + \frac{4}{4}\right) = \boxed{\frac{3\pi}{4}}$$ /

$$\int \left(\frac{1}{x}\right)^2 dx = \int \frac{1}{x^2}\,dx = \frac{x^{-2+1}}{-2+1} + C = \frac{x^{-1}}{-1} + C$$

4.) Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.

$y = e^x$, $y = 0$, $x = -1$, $x = 1$; about the x-axis.



X	$y = e^x$
-1	0.368
0	1
1	2.718

$$\int (e^x)^2 dx = \int e^{2x} dx$$

$$A(x) = \pi y^2 = \pi (e^x)^2$$

$$V = \int_{-1}^{1} A(x)\,dx = \pi \int_{-1}^{1} (e^x)^2 \, dx$$

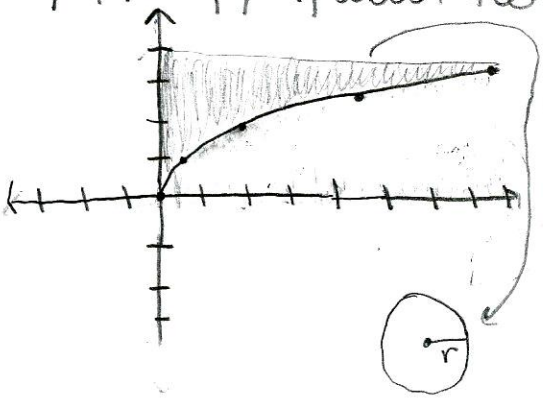
$$V = \pi \left[\frac{e^{2x}}{2} \right]_{-1}^{1}$$

$$V = \pi \left[\frac{e^2}{2} - \frac{e^{-2}}{2} \right] = \frac{\pi}{2}\left(e^2 - \frac{1}{e^2}\right)$$

$$V \approx 3.63\pi = \boxed{11.39}$$

6.) Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.

$2x = y^2$, $x = 0$, $y = 4$; about the y-axis.



$X = \frac{y^2}{2}$	Y
0	0
$\frac{1}{2}$	1
2	2
4.5	3
8	4

$$A(y) = \pi x^2 = \pi \left(\frac{y^2}{2}\right)^2$$

$$V = \int_0^4 A(y)\,dy = \pi \int_0^4 \left(\frac{y^2}{2}\right)^2 dy$$

$$= \pi \int_0^4 \frac{y^4}{4}\,dy \checkmark$$

$$= \pi \left[\frac{y^5}{20} \right]_0^4$$

$$V = \pi \left[\frac{4^5}{20} - \frac{0^5}{20} \right] = \frac{1024}{20}\pi$$

$$= \boxed{\frac{256\pi}{5}} \checkmark$$

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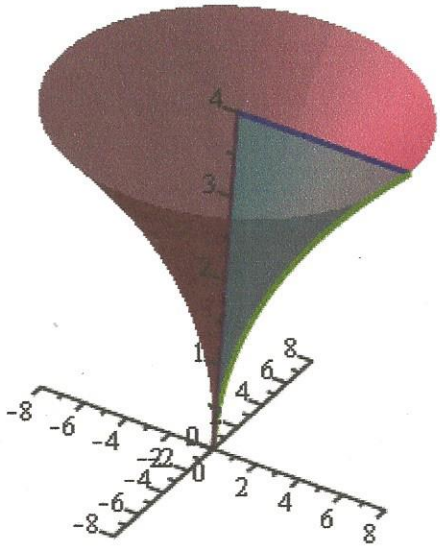
```
> VolumeOfRevolution(4, g(x), x = 0 ..8, output = plot, showregion = true, orientation = [
      -43, 56, 18 ], axis = vertical)
```



The solid of revolution created on $0 \le x \le 8$ by rotation of $f(x) = 4$ and $g(x) = \sqrt{2}\,\sqrt{x}$ about the axis $x = 0$. The slice that is rotated is shaded in burgundy.

```
> VolumeOfRevolution(4, g(x), x = 0 ..8, output = value, axis = vertical)
```

$$\frac{256}{5}\,\pi \qquad \textbf{(2)}$$

***** END *****

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10.) Evaluate the integral. $\int \ln\sqrt{x}\,dx$

$$\int \ln\sqrt{x}\,dx = \int \ln x^{\frac{1}{2}}\,dx = \frac{1}{2}\int \ln(x)\,dx$$

$u = \ln x \quad du = \frac{1}{x}dx \quad dv = dx \quad v = x$ (+1, −1)

$$\int \ln\sqrt{x}\cdot dx = \frac{1}{2}\left(x\cdot\ln x - \int x\cdot\frac{1}{x}\,dx\right)$$

$$= \frac{1}{2}(x\ln x - x) + C$$

$$= \frac{x}{2}(\ln x - 1) + C$$

12.) Evaluate the integral. $\int \tan^{-1} 2y\,dy$

$u = \tan^{-1} 2y \quad dv = dy$

$du = \frac{2}{1+(2y)^2}dy \quad v = y$ (+1, −1)

$$\int \tan^{-1} 2y\,dy = \tan^{-1} 2y\cdot y - \int y\cdot\frac{2}{1+(2y)^2}\,dy$$

$$\int \tan^{-1} 2y\cdot dy = y\tan^{-1}(2y) - \int \frac{2y}{1+4y^2}\,dy$$

$$= y\tan^{-1}(2y) - \frac{\ln(1+4y^2)}{4} + C$$