NAME ______________________ DATE ______________

12-2 Completing The Square

Objective: To solve quadratic equations by completing the square.

Method of Completing the Square

A method of transforming a quadratic equation so that it is in the form

$$\text{perfect square} = k \quad (k \geq 0).$$

For $x^2 + bx + \underline{?}$:

1. Find half the coefficient of x: $\frac{b}{2}$
2. Square the result of Step 1: $\left(\frac{b}{2}\right)^2$
3. Add the result of Step 2 to $x^2 + bx$: $x^2 + bx + \left(\frac{b}{2}\right)^2$
4. You have completed the square: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

Example 1 Complete the square:

a. $x^2 + 6x + \underline{?}$

b. $x^2 - 4x + \underline{?}$

Solution

a. $\underbrace{x^2 + 6x}_{\left(\frac{6}{2}\right)^2 = 9} + 9 = (x + 3)^2$

b. $\underbrace{x^2 - 4x}_{\left(-\frac{4}{2}\right)^2 = 4} + 4 = (x - 2)^2$

Complete the square.

1. $x^2 - 8x + \underline{?}$
2. $x^2 - 10x + \underline{?}$
3. $v^2 - 20v + \underline{?}$
4. $c^2 - 12c + \underline{?}$
5. $x^2 - 18x + \underline{?}$
6. $x^2 + 2x + \underline{?}$

Example 2 Solve $x^2 + 3x - 10 = 0$ by completing the square.

Solution

$x^2 + 3x = 10$ — Move the constant term to the right side.

$x^2 + 3x + \left(\frac{3}{2}\right)^2 = 10 + \left(\frac{3}{2}\right)^2$ — Half of the coefficient of x is $\frac{3}{2}$.

$x^2 + 3x + \frac{9}{4} = 10 + \frac{9}{4}$ — Square it and add the result to *both* sides.

perfect square $\longrightarrow \left(x + \frac{3}{2}\right)^2 = \frac{49}{4} \longleftarrow$ constant

$$x + \frac{3}{2} = \pm\frac{7}{2}$$

$$x = -\frac{3}{2} \pm \frac{7}{2}$$

$$= \frac{-3 \pm 7}{2}$$

$$x = 2 \quad \text{or} \quad x = -5$$

The check is left for you. The solution set is $\{-5, 2\}$.

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12–2 Completing the Square *(continued)*

Solve by completing the square.

7. $x^2 + 8x = -15$	**8.** $x^2 + 6x = -8$	**9.** $x^2 + 4x = 5$
10. $x^2 - 7x - 8 = 0$	**11.** $x^2 - 3x - 4 = 0$	**12.** $x^2 + x - 12 = 0$
13. $y^2 + 7y = -12$	**14.** $n^2 - 5n = 36$	**15.** $x^2 + 3x = 10$
16. $y^2 - 9y + 8 = 0$	**17.** $x^2 - 5x - 50 = 0$	**18.** $x^2 - 3x - 40 = 0$

Example 3 Solve $3x^2 - 5x - 2 = 0$ by completing the square.

Solution

$$3x^2 - 5x = 2$$ Move the constant term to the right side.

$$x^2 - \frac{5}{3}x = \frac{2}{3}$$ Divide both sides by 3 so that the coefficient of x^2 will be 1.

$$x^2 - \frac{5}{3}x + \left(\frac{5}{6}\right)^2 = \frac{2}{3} + \left(\frac{5}{6}\right)^2$$ Complete the square.

$$x^2 - \frac{5}{3}x + \frac{25}{36} = \frac{2}{3} + \frac{25}{36}$$

$$\left(x - \frac{5}{6}\right)^2 = \frac{49}{36}$$

$$x - \frac{5}{6} = \pm\frac{7}{6}$$

$$x = \frac{5}{6} \pm \frac{7}{6}$$

$$x = 2 \quad \text{or} \quad x = -\frac{1}{3}$$

The check is left for you. The solution set is $\left\{2, -\frac{1}{3}\right\}$.

Solve by completing the square.

19. $2n^2 - n - 6 = 0$	**20.** $2x^2 + x - 1 = 0$	**21.** $2x^2 - 5x = 12$
22. $2x^2 - 5x - 3 = 0$	**23.** $3x^2 + 11x = 4$	**24.** $5x^2 + 4x - 1 = 0$

Solve the equations by (a) completing the square and (b) by factoring.

25. $x^2 - 12x + 35 = 0$	**26.** $x^2 + 18x + 45 = 0$	**27.** $a^2 + 17a - 60 = 0$
28. $3x^2 - 5x = 2$	**29.** $2x^2 + x = 10$	**30.** $2x^2 - 11x = -5$

Solve. Write irrational roots in simplest radical form.

31. $y^2 + 8y = -10$	**32.** $4x^2 - 4x = 7$	**33.** $3x^2 - 6x - 2 = 0$

Mixed Review Exercises

Simplify.

1. $\sqrt{140}$ **2.** $\sqrt{1200}$ **3.** $\sqrt{16x^4y^7}$ **4.** $\sqrt{60a^6b^5c^3}$

5. $\frac{x}{3 - x} + \frac{4}{x + 3}$ **6.** $1 + \frac{n^2}{n^2 - 1}$ **7.** $2(3x - 1) + (x + 5)(3x + 1)$