

## 5-11 Using Several Methods of Factoring

**Objective:** To factor polynomials completely.

### Vocabulary

**Factored completely** A polynomial expressed as the product of a monomial and one or more prime polynomials, that is when it cannot be factored further.

### Guidelines for Factoring Completely

- Factor out the greatest monomial factor first.
- Look for a difference of squares.  
Pattern:  $a^2 - b^2 = (a - b)(a + b)$  (However,  $a^2 + b^2$  can't be factored.)
- Look for a perfect square trinomial.  
Patterns:  $(a + b)^2 = a^2 + 2ab + b^2$   $(a - b)^2 = a^2 - 2ab + b^2$
- If the trinomial is not a perfect square, look for a pair of binomial factors.
- If a polynomial has four or more terms, look for a way to group the terms in pairs or in a group of three terms that is a perfect square trinomial.
- Make sure that each binomial or trinomial factor is prime.
- Check your work by multiplying the factors.

**Example 1** Factor  $8x^3 - 512x$  completely.

**Solution**

$$8x^3 - 512x = 8x(x^2 - 64)$$

Greatest monomial factor  $\uparrow$   $\leftarrow$  Difference of squares

$$= 8x(x + 8)(x - 8)$$

**Example 2** Factor  $3x^3 + 3x^2 - 18x$  completely.

**Solution**

$$3x^3 + 3x^2 - 18x = 3x(x^2 + x - 6)$$

Greatest monomial factor  $\uparrow$   $\leftarrow$  Trinomial

$$= 3x(x + 3)(x - 2)$$

**Factor completely.**

- |                           |                           |
|---------------------------|---------------------------|
| 1. $3x^3 - 12x$           | 2. $5m^3 - 45m$           |
| 3. $3a^2 + 6ab + 3b^2$    | 4. $-x^3 + 4xy^2$         |
| 5. $-12z^3 + 30z^2 + 18z$ | 6. $16r^4 - 24r^3 + 9r^2$ |
| 7. $20x^3 - 28x^2 + 8x$   | 8. $t^3 + t^2 - 2t$       |
| 9. $2x^2 - 128$           | 10. $2x^4 - 162$          |
| 11. $25z^3 - 36y^2z$      | 12. $6x^2 + 22xy - 8y^2$  |

**5-11 Using Several Methods of Factoring** (continued)

**Example 3** Factor  $5a^2b^3 + 2a^3b^2 - 3ab^4$  completely.

**Solution** First rewrite the polynomial in order of decreasing degree in  $a$ .

$$\begin{aligned} 5a^2b^3 + 2a^3b^2 - 3ab^4 &= 2a^3b^2 + 5a^2b^3 - 3ab^4 \\ &= ab^2(2a^2 + 5ab - 3b^2) \\ \text{Greatest monomial factor } \uparrow & \quad \quad \quad \leftarrow \text{Trinomial} \\ &= ab^2(2a - b)(a + 3b) \end{aligned}$$

**Example 4** Factor  $a^2b - 4b + 3a^2 - 12$  completely.

$$\begin{aligned} \text{Solution} \quad a^2b - 4b + 3a^2 - 12 &= b(a^2 - 4) + 3(a^2 - 4) && \text{Group and factor.} \\ &= (b + 3)(a^2 - 4) && \text{Use the distributive property.} \\ & \quad \quad \quad \uparrow \quad \quad \quad \leftarrow \text{Difference of squares} \\ &= (b + 3)(a + 2)(a - 2) \end{aligned}$$

**Factor completely.**

13.  $a^3x - 9ax^3$

14.  $18x^3 - 24x^2 + 8x$

15.  $20 - 60x + 45x^2$

16.  $6x^2 - 18xy + 12y^2$

17.  $9x^3 + 108x + 63x^2$

18.  $10k^3 + 25k - 35k^2$

19.  $32r^4 - 48r^3 + 18r^2$

20.  $12ab - 3b^2 - 12a^2$

21.  $bc^2 - 4b + 3c^2 - 12$

22.  $x^3 - x + 6x^2 - 6$

23.  $x^2 + 6xy + 9y^2 - 16$

24.  $a^3 + a^2b - ab^2 - b^3$

25.  $y^4 - 9y^2 + 20$

26.  $x^4 - 10x^2 + 9$

27.  $x^4 - 13x^2 + 36$

28.  $x^4 - 24x^2 + 144$

29.  $b^4 - 8b^2 + 16$

30.  $a^3 + 2a^2 - 5a - 10$

**Mixed Review Exercises**

**Simplify.**

1.  $\left(-\frac{1}{3}\right)\left(\frac{1}{4}\right)(60)$

2.  $\frac{1}{8}(56)$

3.  $-\frac{1}{7}(56)\left(-\frac{1}{8}\right)$

4.  $\frac{120b}{8}$

5.  $45 \div \left(\frac{1}{5}\right)$

6.  $600 \div (-5)$

**Factor.**

7.  $x^2 - 11x + 30$

8.  $x^2 + 2x - 35$

9.  $x^2 - x - 20$

10.  $2n^2 + 15n + 7$

11.  $3x^2 + 7x + 4$

12.  $(2x - 6) - 3n(3 - x)$