NAME ______________________ DATE ______________

4 Polynomials

4–1 Exponents

Objective: To write and simplify expressions involving exponents.

Vocabulary

Power of a number A product of equal factors. For example, $2 \times 2 \times 2$, or 2^3, is the third power of 2.

Base of a power The number that is used as a factor. For example, 2 is the base in 2^3.

Exponent In a power, the number that indicates how many times the base is used as a factor. For example, 3 is the exponent in 2^3.

Exponential form The expression 2^3 is the exponential form of $2 \cdot 2 \cdot 2$.

CAUTION 1 Be careful when an expression contains both parentheses and exponents.
$(3y)^2$ means $(3y)(3y)$. 2 is the exponent of the base $3y$.
$3y^2$ means $3 \cdot y \cdot y$. 2 is the exponent of the base y.

CAUTION 2 Follow the correct order when simplifying expressions.
1. First simplify expressions within grouping symbols.
2. Then simplify powers.
3. Then simplify products and quotients in order from left to right.
4. Then simplify sums and differences in order from left to right.

Example 1 Write each expression in exponential form.
a. $5 \cdot 5 \cdot 5$ **b.** $a \cdot a \cdot a \cdot a \cdot a$ **c.** $-2 \cdot x \cdot y \cdot 5 \cdot x \cdot x$

Solution **a.** 5^3 **b.** a^5 **c.** $-10x^3y$

Write each expression in exponential form.

1. $x \cdot x \cdot x \cdot x$ **x^4**
2. $m \cdot m$ **m^2**
3. $3 \cdot t \cdot t \cdot t \cdot t$ **$3t^4$**
4. $c \cdot e \cdot 2 \cdot c \cdot e$ **$2c^2e^2$**
5. $-4 \cdot z \cdot z \cdot z$ **$-4z^3$**
6. $y \cdot y \cdot (-2)$ **$-2y^2$**
7. $-2 \cdot x \cdot x \cdot 3 \cdot x$ **$-6x^3$**
8. $5 \cdot n \cdot (-3)$ **$-15n$**
9. $a \cdot a \cdot a \cdot b \cdot b$ **a^3b^2**
10. $c \cdot c \cdot d \cdot d \cdot d \cdot c$ **c^3d^3**
11. $m \cdot 6 \cdot n \cdot m$ **$6m^2n$**
12. $u \cdot v \cdot u \cdot v \cdot 8$ **$8u^2v^2$**

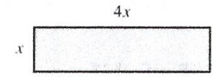
Example 2 Evaluate x^3 if $x = -2$.

Solution
$x^3 = (-2)^3$ Replace x with -2.
$= (-2)(-2)(-2)$ Simplify.
$= -8$

13. Evaluate x^4 if $x = -2$. **16**
14. Evaluate x^3 if $x = -3$. **-27**
15. Evaluate y^5 if $y = -1$. **-1**
16. Evaluate x^2 if $x = -5$. **25**

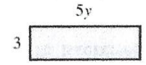
4–1 Exponents *(continued)*

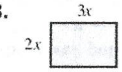
Example 3 Find the area of the rectangle.

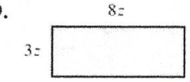


Solution Area = length × width
$= 4x \cdot x$
$= 4x^2$

Find the area of each rectangle. **17. $15y$ 18. $6x^2$ 19. $24z^2$**

17. 

18. 

19. 

Example 4 Simplify: **a.** -2^4 **b.** $(-2)^4$ **c.** $(1+2)^3$ **d.** $1+2^3$

Solution
a. $-2^4 = -(2 \cdot 2 \cdot 2 \cdot 2) = -16$ **b.** $(-2)^4 = (-2)(-2)(-2)(-2) = 16$
c. $(1+2)^3 = 3^3 = 27$ **d.** $1 + 2^3 = 1 + 2 \cdot 2 \cdot 2 = 1 + 8 = 9$

Simplify.

20. **a.** 3^4 **81** **b.** 4^3 **64**
21. **a.** -3^2 **-9** **b.** $(-3)^2$ **9**
22. **a.** -2^3 **-8** **b.** $(-2)^3$ **-8**
23. **a.** $2 \cdot 4^2$ **32** **b.** $(2 \cdot 4)^2$ **64**
24. **a.** $5 - 2^2$ **1** **b.** $(5-2)^2$ **9**
25. **a.** $2 - 5^2$ **-23** **b.** $(2-5)^2$ **9**
26. **a.** $5 - 3^2$ **-4** **b.** $(5-3)^2$ **4**
27. **a.** $2 \cdot 5 - 3^2$ **1** **b.** $(2 \cdot 5 - 3)^2$ **49**

Example 5 Evaluate $(2a - b)^2$ if $a = 2$ and $b = -3$.

Solution
$(2a - b)^2 = [2 \cdot 2 - (-3)]^2$ Replace a with 2 and b with -3.
$= [4 + 3]^2$ Simplify the expression within the brackets.
$= 7^2$
$= 49$

Evaluate each expression if $x = 2$ and $y = -1$.

28. **a.** $2x + y^2$ **5** **b.** $(2x + y)^2$ **9**
29. **a.** $2 + xy^2$ **4** **b.** $(2 + xy)^2$ **0**
30. **a.** $2x + y^3$ **3** **b.** $(2x + y)^3$ **27**
31. **a.** $(x + 2y)^3$ **0** **b.** $x^3 + 2y^3$ **6**
32. **a.** $2x - y^2$ **3** **b.** $(2x - y)^2$ **25**
33. **a.** $2 - xy^2$ **0** **b.** $(2 - xy)^2$ **16**
34. **a.** $2x - y^3$ **5** **b.** $(2x - y)^3$ **125**
35. **a.** $(x - 2y)^3$ **64** **b.** $x^3 - 2y^3$ **10**

Mixed Review Exercises

Solve.

1. $-6x = 42$ **$\{-7\}$**
2. $2(n - 3) = 24$ **$\{15\}$**
3. $24 = -8x$ **$\{-3\}$**
4. $-n + 6 = 8$ **$\{-2\}$**
5. $x - 5 = -3$ **$\{2\}$**
6. $-y + 10 = 6$ **$\{4\}$**
7. $-\frac{1}{2}(x + 3) = 4$ **$\{-11\}$**
8. $\frac{1}{3}x = 10$ **$\{30\}$**

NAME ____________________ DATE ____________

4–2 Adding and Subtracting Polynomials

Objective: To add and subtract polynomials.

Vocabulary

Monomial An expression that is either a numeral, a variable, or the product of a numeral and one or more variables. For example: 13, m, $8c$, $2xy$, $5p^2q$.

Coefficient In the monomial $9x^2y^3$, 9 is the coefficient, or numerical coefficient.

Similar, or like, terms Two monomials that are exactly alike or the same except for their numerical coefficient. For example, $-3xy$ and $7xy$ are similar.

Constant monomial or constant A numeral such as 7.

Polynomial A sum of monomials. For example, $x^2 + 3x + y^2 + 2$.

Binomial A polynomial of only two terms. For example, $2x + 5$.

Trinomial A polynomial of only three terms. For example, $a^2 + 2ab + b^2$.

Simplified form, or simplest form, of a polynomial A polynomial which has no two of its terms similar.

CAUTION When a monomial does not have a written numerical coefficient, remember that its coefficient is 1. For example, $x^6y^2 = 1x^6y^2$.

Example 1 Simplify $-5x^3 + 2x^2 + x^2 + 7x^3 - 4$.

Solution

$$\begin{aligned} -5x^3 + 2x^2 + x^2 + 7x^3 - 4 &= (-5x^3 + 7x^3) + (2x^2 + x^2) - 4 \\ &= (-5 + 7)x^3 + (2 + 1)x^2 - 4 \\ &= 2x^3 + 3x^2 - 4 \end{aligned}$$

Simplify.

1. $2x - y + 3x - 2y$ **$5x - 3y$**
2. $7m - 5n - 2m + n$ **$5m - 4n$**
3. $4x^2 - 3x - 2x^2 + 7x - 2$ **$2x^2 + 4x - 2$**
4. $n^2 - 3n - 5n^2 + 7n + 6n^2$ **$2n^2 + 4n$**
5. $a^2 + 2ab - 5ab + 4a^2$ **$5a^2 - 3ab$**
6. $x^2y - y^3 - 8x^2y + 5y^3$ **$-7x^2y + 4y^3$**
7. $a^2b - 2ab^2 + 5a^3 - 3a^2b$ **$5a^3 - 2a^2b - 2ab^2$**
8. $-6xy^2 + 5x^2y - x^3 + xy^2 + 3x^3 - 2x^2y$ **$2x^3 + 3x^2y - 5xy^2$**

Example 2 Add $2x^2 + 5xy - 6y^2$ and $8x^2 + 6xy + y^2$.

Solution 1 First group similar terms and then combine them.

$$\begin{aligned} (2x^2 + 5xy - 6y^2) + (8x^2 + 6xy + y^2) &= (2x^2 + 8x^2) + (5xy + 6xy) + (-6y^2 + y^2) \\ &= 10x^2 + 11xy - 5y^2 \end{aligned}$$

Solution 2

$$\begin{array}{r} 2x^2 + 5xy - 6y^2 \\ 8x^2 + 6xy + y^2 \\ \hline 10x^2 + 11xy - 5y^2 \end{array}$$

You can also align similar terms vertically and add.

NAME ____________________ DATE ____________

4–2 Adding and Subtracting Polynomials *(continued)*

Vocabulary

Degree of a variable in a monomial The number of times that the variable occurs as a factor in a monomial. For example, in $7x^3y$, the degree of x is 3, and the degree of y is 1.

Degree of a monomial The sum of the degrees of its variables. For example, the degree of $8x^2y^3$ is 5. The degree of any nonzero constant monomial, such as 10, is 0.

Degree of a polynomial The greatest of the degrees of its terms after it has been simplified. For example, the degree of $-5x^3 + 2x^2 + x^2 + 5x^3 - 4$ is 2, since it can be simplified to $3x^2 - 4$.

Add.

9. $\begin{array}{r} 3a - 1 \\ 4a + 3 \\ \hline \mathbf{7a + 2} \end{array}$	10. $\begin{array}{r} 4n + 2 \\ -2n - 5 \\ \hline \mathbf{2n - 3} \end{array}$	11. $\begin{array}{r} 2x - 3y \\ -2x + 6y \\ \hline \mathbf{3y} \end{array}$	12. $\begin{array}{r} 5n - 2p \\ -3n + 5p \\ \hline \mathbf{2n + 3p} \end{array}$
13. $\begin{array}{r} 4x - 5y + 3 \\ -2x + 7y + 7 \\ \hline \mathbf{2x + 2y + 10} \end{array}$	14. $\begin{array}{r} 2a - 3b - 6 \\ 3a - b + 8 \\ \hline \mathbf{5a - 4b + 2} \end{array}$	15. $\begin{array}{r} 6x^2 - 3x + 2 \\ 2x^2 + x - 5 \\ \hline \mathbf{8x^2 - 2x - 3} \end{array}$	16. $\begin{array}{r} 5 - 2n - 6n^2 \\ -3 + n - 2n^2 \\ \hline \mathbf{2 - n - 8n^2} \end{array}$
17. $\begin{array}{r} 4c^2 - 3cd - 5d^2 \\ -c^2 + 6cd - 2d^2 \\ \hline \mathbf{3c^2 + 3cd - 7d^2} \end{array}$	18. $\begin{array}{r} 6a^2 - 2ab \\ -2a^2 + 5ab - b^2 \\ \hline \mathbf{4a^2 + 3ab - b^2} \end{array}$	19. $\begin{array}{r} 3x - 2y - 5z + 1 \\ 2x + y - 3z \\ 3y + z + 3 \\ \hline \mathbf{5x + 2y - 7z + 4} \end{array}$	20. $\begin{array}{r} 6a - 2b + 4 \\ 3a - 5c - 1 \\ -a - b + 6c + 5 \\ \hline \mathbf{8a - 3b + c + 8} \end{array}$

Example 3 Subtract $-x^2 + 5xy + 6y^2 - 3$ from $3x^2 - 6xy - 2y^2 - 5$.

Solution 1 Add the opposite of $(-x^2 + 5xy + 6y^2 - 3)$ to $3x^2 - 6xy - 2y^2 - 5$.

$$\begin{aligned} &(3x^2 - 6xy - 2y^2 - 5) - (-x^2 + 5xy + 6y^2 - 3) = \\ &(3x^2 - 6xy - 2y^2 - 5) + (x^2 - 5xy - 6y^2 + 3) = 4x^2 - 11xy - 8y^2 - 2 \end{aligned}$$

Solution 2 You can also align similar terms vertically.

$$\begin{array}{r} 3x^2 - 6xy - 2y^2 - 5 \\ -(-x^2 + 5xy + 6y^2 - 3) \\ \hline \end{array} \longrightarrow \text{Change to the opposite signs and add.} \longrightarrow \begin{array}{r} 3x^2 - 6xy - 2y^2 - 5 \\ x^2 - 5xy - 6y^2 + 3 \\ \hline 4x^2 - 11xy - 8y^2 - 2 \end{array}$$

21–30. In Exercises 9–18, subtract the lower polynomial from the upper one.

21. $-a - 4$ 22. $6n + 7$ 23. $4x - 9y$ 24. $8n - 7p$ 25. $6x - 12y - 4$ 26. $-a - 2b - 14$ 27. $4x^2 - 4x + 7$ 28. $8 - 3n - 4n^2$ 29. $5c^2 - 9cd - 3d^2$ 30. $8a^2 - 7ab + b^2$

Mixed Review Exercises

Simplify.

1. -2^3 **-8**
2. $(-3)^2$ **9**
3. $2^2 + 3^2$ **13**
4. $(2 + 3)^2$ **25**

Solve.

5. $3(y + 2) - 2 = 2(4 - y)$ **$\left\{\frac{4}{5}\right\}$**
6. $10 = 2(n + 3)$ **$\{2\}$**
7. $4(x - 10) = 13 - 3(2x + 1)$ **$\{5\}$**
8. $-\frac{2}{5}(n + 3) = 10$ **$\{-28\}$**
9. $c - 2 = |1 - 8|$ **$\{9\}$**
10. $\frac{3}{4}(2y - 6) = y - 7$ **$\{-5\}$**

NAME ____________________ DATE ____________

4–3 Multiplying Monomials

Objective: To multiply monomials.

Rule of Exponents for Products of Powers	Example
For all positive integers m and n: $a^m \cdot a^n = a^{m+n}$. This means that when you multiply two powers having the same base, you add the exponents.	$x^3 \cdot x^5 = x^{3+5} = x^8$

CAUTION Use the rule of exponents for products of powers only when the two powers to be multiplied have the *same base.* For example,

$$m^2 \cdot n^3 = m^2n^3, \text{ not } mn^5$$

Example 1 Simplify: a. $x^2 \cdot x^5$ b. $c^6 \cdot c^3$ c. $a \cdot a^5$

Solution a. $x^2 \cdot x^5 = x^{2+5} = x^7$
b. $c^6 \cdot c^3 = c^{6+3} = c^9$
c. $a \cdot a^5 = a^1 \cdot a^5 = a^{1+5} = a^6$

Simplify.

1. $x^3 \cdot x^6$ $\mathbf{x^9}$
2. $c^7 \cdot c^8$ $\mathbf{c^{15}}$
3. $a^9 \cdot a^{10}$ $\mathbf{a^{19}}$
4. $x^2 \cdot x^3 \cdot x$ $\mathbf{x^6}$
5. $n^2 \cdot n^2 \cdot n^3$ $\mathbf{n^7}$
6. $c \cdot c^5 \cdot c^2$ $\mathbf{c^8}$
7. $a^2 \cdot a^3 \cdot a^5$ $\mathbf{a^{10}}$
8. $x^5 \cdot x^6 \cdot x^7$ $\mathbf{x^{18}}$
9. $c^3 \cdot c^6 \cdot c^7$ $\mathbf{c^{16}}$
10. $m^2 \cdot m^6 \cdot m^8$ $\mathbf{m^{16}}$
11. $n^{10} \cdot n \cdot n^3$ $\mathbf{n^{14}}$
12. $x \cdot x^9 \cdot x^{10}$ $\mathbf{x^{20}}$

Example 2 Simplify $(2x^3)(4x^4)$.

Solution
$(2x^3)(4x^4) = (2 \cdot 4)(x^3 \cdot x^4)$ Use the commutative and associative
$= 8(x^{3+4})$ properties of multiplication.
$= 8x^7$ Use the rule of exponents for products of powers.

Simplify.

13. $(2a^4)(5a^3)$ $\mathbf{10a^7}$
14. $(4x^3)(3x^4)$ $\mathbf{12x^7}$
15. $(7m^5)(2m^6)$ $\mathbf{14m^{11}}$
16. $(5x^4)(3x^2)$ $\mathbf{15x^6}$
17. $(-2xy^2)(-3x^2y)$ $\mathbf{6x^3y^3}$
18. $(4a^2b)(-3ab^3)$ $\mathbf{-12a^3b^4}$
19. $(3ab)(a^2b)(5b^2)$ $\mathbf{15a^3b^4}$
20. $(6x^2y)(2xy^2)(3x)$ $\mathbf{36x^4y^3}$
21. $(3cd^4)(-2c^2)(4cd^2)$ $\mathbf{-24c^4d^6}$
22. $(5a^3b^2)(-4a^2b^2)(-2ab^3)$ $\mathbf{40a^6b^7}$
23. $(-x^2y^2)(3x^2y)(-4xy^3)$ $\mathbf{12x^5y^6}$
24. $(-a^2b)(-5ab^3)(-b^2)$ $\mathbf{-5a^3b^6}$

4–3 Multiplying Monomials *(continued)*

Example 3 Simplify $\left(\frac{10x^2y}{3}\right)\left(\frac{6x^3y^2}{5}\right)$.

Solution $\left(\frac{10x^2y}{3}\right)\left(\frac{6x^3y^2}{5}\right) = \left(\frac{\overset{2}{\cancel{10}}}{\underset{1}{\cancel{3}}} \cdot \frac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{5}}}\right)(x^2 \cdot x^3)(y \cdot y^2) = 4x^5y^3$

Simplify.

25. $\left(\frac{3}{4}r^2\right)\left(\frac{4}{3}t^2\right)$ $\mathbf{r^2t^2}$
26. $\left(\frac{6h^2k^3}{5}\right)\left(\frac{20hk^2}{3}\right)$ $\mathbf{8h^3k^5}$
27. $(8a)\left(\frac{3}{4}a^2\right)$ $\mathbf{6a^3}$
28. $(12c^2)\left(-\frac{5}{6}cd^2\right)$ $\mathbf{-10c^3d^2}$
29. $\left(\frac{3a^2}{7}\right)(35a^5)$ $\mathbf{15a^7}$
30. $\left(\frac{8x^2y}{5}\right)\left(\frac{15xy^2}{16}\right)$ $\mathbf{\frac{3}{2}x^3y^3}$
31. $\left(-\frac{5}{6}x^3\right)(3xy^2)(-y^2)$ $\mathbf{\frac{5}{2}x^4y^4}$
32. $(3y^2)\left(\frac{2}{3}y^2\right)\left(\frac{1}{2}y\right)$ $\mathbf{y^5}$

Example 4 Simplify $(2x^3)(-4x^2) + (5x^2)(3x^3)$.

Solution
$(2x^3)(-4x^2) + (5x^2)(3x^3) = (2)(-4)(x^3 \cdot x^2) + (5 \cdot 3)(x^2 \cdot x^3)$
$= -8x^5 + 15x^5$
$= 7x^5$

Simplify.

33. $(2x)(3x^3) + (5x^2)(4x^2)$ $\mathbf{26x^4}$
34. $(3x^5)(4x) - (5x^3)(2x^3)$ $\mathbf{2x^6}$
35. $(6x^2)(2x^3) + (3x)(5x^4)$ $\mathbf{27x^5}$
36. $(6x^5)(4x^2) - (2x^3)(12x^4)$ $\mathbf{0}$
37. $(3a^4)(-2a^3) + (2a^2)(a^5)$ $\mathbf{-4a^7}$
38. $(4y^2)(4y) - (5y^2)(3y)$ $\mathbf{y^3}$

Mixed Review Exercises

Simplify.

1. $3 + 4^2$ **19**
2. $(3 + 4)^2$ **49**
3. $3a^2 + 5b^2 - a^2 - 2b^2$ $\mathbf{2a^2 + 3b^2}$
4. $2 \cdot 5^2$ **50**
5. $(2 \cdot 5)^2$ **100**
6. $2x^2 - 3x + 4 + 5x + 3x^2$ $\mathbf{5x^2 + 2x + 4}$

Solve.

7. $3(y + 2) = 24$ **{6}**
8. $10z = 20 + 5z$ **{4}**
9. $6n - 12 = 2n$ **{3}**
10. $\frac{n}{4} + 2 = 5$ **{12}**
11. $3(x - 2) = 9$ **{5}**
12. $\frac{x}{3} - 1 = 2$ **{9}**

NAME ______________________ DATE ______________

4–4 Powers of Monomials

Objective: To find powers of monomials.

Rules of Exponents	Examples
Rule of Exponents for a Power of a Power For all positive integers m and n: $(a^m)^n = a^{mn}$. To find a power of a power, you multiply the exponents.	$(2^3)^4 = 2^{3 \cdot 4} = 2^{12}$
Rule of Exponents for a Power of a Product For every positive integer m: $(ab)^m = a^m b^m$. To find a power of a product, you find the power of each factor and then multiply.	$(-2x)^5 = (-2)^5(x)^5 = -32x^5$

CAUTION $(x^7)^6 = x^{7 \cdot 6} = x^{42}$ but $x^7 \cdot x^6 = x^{7+6} = x^{13}$

Example 1 Simplify: a. $(x^2)^4$ b. $(u^3)^5$

Solution Use the rule for a power of a power.

a. $(x^2)^4 = x^{2 \cdot 4} = x^8$ b. $(u^3)^5 = u^{3 \cdot 5} = u^{15}$

Simplify.

1. $(a^2)^3$ $\mathbf{a^6}$
2. $(x^4)^3$ $\mathbf{x^{12}}$
3. $(t^5)^3$ $\mathbf{t^{15}}$
4. $(c^3)^3$ $\mathbf{c^9}$
5. $(t^2)^3$ $\mathbf{t^6}$
6. $(x^5)^2$ $\mathbf{x^{10}}$
7. $(y^{10})^3$ $\mathbf{y^{30}}$
8. $(a^7)^8$ $\mathbf{a^{56}}$

Example 2 Simplify: a. $(2x)^4$ b. $(-6k)^3$

Solution Use the rule for a power of a product.

a. $(2x)^4 = 2^4 \cdot x^4 = 16x^4$ b. $(-6k)^3 = (-6)^3 \cdot k^3 = -216k^3$

Simplify.

9. $(5a)^2$ $\mathbf{25a^2}$
10. $(-6x)^2$ $\mathbf{36x^2}$
11. $(-3t)^3$ $\mathbf{-27t^3}$
12. $(-4c)^2$ $\mathbf{16c^2}$
13. $(-5x)^3$ $\mathbf{-125x^3}$
14. $(-4t)^3$ $\mathbf{-64t^3}$
15. $(-2t)^4$ $\mathbf{16t^4}$
16. $(6x)^3$ $\mathbf{216x^3}$
17. $(5x)^4$ $\mathbf{625x^4}$
18. $(7n)^2$ $\mathbf{49n^2}$
19. $\left(\frac{1}{2}a\right)^2$ $\mathbf{\frac{1}{4}a^2}$
20. $\left(-\frac{1}{3}a\right)^3$ $\mathbf{-\frac{1}{27}a^3}$

NAME ______________________ DATE ______________

Example 3 Evaluate if $x = 3$: a. $2x^3$ b. $(2x)^3$ c. $2^3 \cdot x^3$

Solution

a. $2x^3 = 2(3)^3 = 2(27) = 54$

b. $(2x)^3 = (2 \cdot 3)^3 = 6^3 = 216$

c. $2^3 \cdot x^3 = 2^3 \cdot 3^3 = 8 \cdot 27 = 216$

Evaluate if $x = 2$ and $y = 4$.

21. a. $2x^3$ **16** b. $(2x)^3$ **64** c. $2^3 \cdot x^3$ **64**
22. a. $4y^2$ **64** b. $(4y)^2$ **256** c. $4^2 \cdot y^2$ **256**
23. a. x^2y^3 **256** b. x^2y^2 **64** c. $(xy)^2$ **64**
24. a. xy^3 **128** b. $(xy)^3$ **512** c. $x^3 \cdot y^3$ **512**
25. a. $3x^2$ **12** b. $(3x)^2$ **36** c. $3^2 \cdot x^2$ **36**
26. a. $5x^2$ **20** b. $(5x)^2$ **100** c. $5^2 \cdot x^2$ **100**
27. a. xy^2 **32** b. $(x^2y)^2$ **256** c. x^3y **32**
28. a. $2xy$ **16** b. $2x^2y$ **32** c. $2xy^2$ **64**
29. a. $6x^2 \div x$ **12** b. $(6x)^2 \div x$ **72** c. $6(x^2 \div x)$ **12**

Example 4 Simplify $(-2x^2y^3)^4$.

Solution $(-2x^2y^3)^4 = (-2)^4(x^2)^4(y^3)^4 = 16x^8y^{12}$ [First use the rule for a power of a product and then use the rule for a power of a power.]

Simplify.

30. $(3n^2)^3$ $\mathbf{27n^6}$
31. $(6b^4)^2$ $\mathbf{36b^8}$
32. $\left(\frac{1}{3}x^{10}\right)^3$ $\mathbf{\frac{1}{27}x^{30}}$
33. $\left(\frac{1}{2}x^2\right)^4$ $\mathbf{\frac{1}{16}x^8}$
34. $(2ab^2)^3$ $\mathbf{8a^3b^6}$
35. $(-3x^2y^3)^3$ $\mathbf{-27x^6y^9}$
36. $(4x^3y^2)^3$ $\mathbf{64x^9y^6}$
37. $(-2xy^2)^4$ $\mathbf{16x^4y^8}$
38. $(5m^2n^4)^2$ $\mathbf{25m^4n^8}$

Mixed Review Exercises

Simplify.

1. $(2a^2b)(3ab)(5ab^2)$ $\mathbf{30a^4b^4}$
2. $(-xy^2)(2xy)(-3y)$ $\mathbf{6x^2y^4}$
3. $(3x^2y^3)^4$ $\mathbf{81x^8y^{12}}$
4. $\left(\frac{1}{3}t^2\right)\left(\frac{3}{4}t^3\right)$ $\mathbf{\frac{1}{4}t^5}$
5. $5c - 2a - 3c + a$ $\mathbf{2c - a}$
6. $(2x + 3y + 1) + (3x + 2y)$ $\mathbf{5x + 5y + 1}$
7. $3 \cdot 5^2 + 3 \cdot 5$ **90**
8. $-3^2 \cdot 4$ **−36**
9. $(3^3 + 5^2) \div 2^2$ **13**
10. $7 \cdot 3^2 + 6 \cdot 3 + 2$ **83**
11. $\left(\frac{5}{2}t^2\right)\left(\frac{1}{5}t^3\right)$ $\mathbf{\frac{1}{2}t^5}$
12. $(15mn^2)\left(\frac{1}{3}m^2\right)(4n)$ $\mathbf{20m^3n^3}$

NAME ______________________________ DATE ______________

4-5 Multiplying Polynomials by Monomials

Objective: To multiply a polynomial by a monomial.

Example 1 Multiply: $x(x + 4)$

Solution 1 $x(x + 4) = x(x) + x(4)$
$= x^2 + 4x$

Solution 2
$x + 4$
x — Multiply each term by x.
$x^2 + 4x$

Multiply.

1. $3(x - 2)$ **$3x - 6$**
2. $-2(x + 3)$ **$-2x - 6$**
3. $c(c - 4)$ **$c^2 - 4c$**
4. $a(3 - 2a)$ **$3a - 2a^2$**
5. $-2b(3 - 4b)$ **$-6b + 8b^2$**
6. $-3c(4c + 1)$ **$-12c^2 - 3c$**
7. $5y(y + 6)$ **$5y^2 + 30y$**
8. $-z(4 - 5z)$ **$-4z + 5z^2$**

Example 2 Multiply: $-2x(3x^2 - 2x + 1)$

Solution 1 Multiply each term of the polynomial $3x^2 - 2x + 1$ by the monomial $-2x$.

$-2x(3x^2 - 2x + 1) = -2x(3x^2) - 2x(-2x) - 2x(1)$
$= -6x^3 + 4x^2 - 2x$

Solution 2
$3x^2 - 2x + 1$
$-2x$
$-6x^3 + 4x^2 - 2x$

Multiply.

9. $3x(x^2 - x - 2)$ **$3x^3 - 3x^2 - 6x$**
10. $-2x(x^2 - 4x + 5)$ **$-2x^3 + 8x^2 - 10x$**
11. $-4x(2x^2 - 3x - 7)$ **$-8x^3 + 12x^2 + 28x$**
12. $5x^2(x^2 + x - 3)$ **$5x^4 + 5x^3 - 15x^2$**
13. $-6x^2(x^2 - x - 12)$ **$-6x^4 + 6x^3 + 72x^2$**
14. $4x^3(x^2 - 3x - 6)$ **$4x^5 - 12x^4 - 24x^3$**
15. $3a^2 - 4a - 6$
    $2a$
    $6a^3 - 8a^2 - 12a$
16. $4a^2 - 5a - 7$
    $5a$
    $20a^3 - 25a^2 - 35a$
17. $5x^2 - x - 3$
    $2x^2$
    $10x^4 - 2x^3 - 6x^2$
18. $2k^2 - 3k - 5$
    $-4k^3$
    $-8k^5 + 12k^4 + 20k^3$

Example 3 Multiply: $4x^2y(5x^2 - 3xy + 2y^2)$

Solution Multiply each term of the polynomial by $4x^2y$.

$4x^2y(5x^2 - 3xy + 2y^2) = 4x^2y(5x^2) + 4x^2y(-3xy) + 4x^2y(2y^2)$
$= 20x^4y - 12x^3y^2 + 8x^2y^3$

Multiply.

19. $3x^2y(4x^2 - 5xy - 2y^2)$ **$12x^4y - 15x^3y^2 - 6x^2y^3$**
20. $xy^2(x^2 - 4xy - 5y^2)$ **$x^3y^2 - 4x^2y^3 - 5xy^4$**
21. $-2xy(4x^2 - 3xy + y^2)$ **$-8x^3y + 6x^2y^2 - 2xy^3$**
22. $\frac{1}{3}x^2y(6x^2 - 12xy + 9y^2)$ **$2x^4y - 4x^3y^2 + 3x^2y^3$**

NAME ______________________________ DATE ______________

Multiply.

23. $2xy^2(3x^2 - 7xy - 2y^2)$ **$6x^3y^2 - 14x^2y^3 - 4xy^4$**
24. $-4x^3y(x^2 - 3xy - 6y^2)$ **$-4x^5y + 12x^4y^2 + 24x^3y^3$**
25. $5xy(2x^2 - 4xy + y^2)$ **$10x^3y - 20x^2y^2 + 5xy^3$**
26. $\frac{1}{2}x^2y^2(6x^2 - 4xy - 8y^2)$ **$3x^4y^2 - 2x^3y^3 - 4x^2y^4$**

Example 4 Simplify $3n(n + 2) + n(5 - n)$.

Solution
$3n(n + 2) + n(5 - n) = 3n(n) + 3n(2) + n(5) - n(n)$ — Use the distributive property.
$= 3n^2 + 6n + 5n - n^2$
$= 2n^2 + 11n$ — Combine similar terms.

Simplify.

27. $2x(x - 3) + 3x(x + 2)$ **$5x^2$**
28. $4x(3 - 2x) + 5x(x - 1)$ **$-3x^2 + 7x$**
29. $5x^2(2x - 1) - 2x(3x^2 - 4x)$ **$4x^3 + 3x^2$**
30. $3y(4y^2 - 3y) - 2y^2(y + 1)$ **$10y^3 - 11y^2$**
31. $2n^2(4n - 5) - 3n(2n^2 - 7n)$ **$2n^3 + 11n^2$**
32. $2x(5x^2 - 3x) - x^2(x + 6)$ **$9x^3 - 12x^2$**

Example 5 Solve $n(2 - 3n) + 3(n^2 - 4) = 0$.

Solution
$n(2 - 3n) + 3(n^2 - 4) = 0$ — Use the distributive property.
$2n - 3n^2 + 3n^2 - 12 = 0$ — Combine similar terms.
$2n - 12 = 0$ — To undo the subtraction of 12 from $2n$, add 12 to each side. To undo the multiplication of n by 2, divide each side by 2.
$2n = 12$
$n = 6$

The solution set is $\{6\}$.

Solve.

33. $2(x - 1) + 3 = 7$ **$\{3\}$**
34. $3(y - 2) + 1 = 10$ **$\{5\}$**
35. $2(2m - 3) - 3(2m - 1) = 9$ **$\{-6\}$**
36. $4(3a - 1) - 5(1 - a) = 8$ **$\{1\}$**
37. $y(3 - 2y) + 2(y^2 - 6) = 0$ **$\{4\}$**
38. $0 = 3(1 - 2x) - 5(3 - x)$ **$\{-12\}$**
39. $x(2 - 3x) + 3(x^2 - 6) = 0$ **$\{9\}$**
40. $2x(1 - 3x) + 6(x^2 - 2) = 0$ **$\{6\}$**

Mixed Review Exercises

Simplify.

1. $(2xy^2)^3$ **$8x^3y^6$**
2. $(-4a^4b^3)^2$ **$16a^8b^6$**
3. $(-2n)^4$ **$16n^4$**
4. $(2a^2b)^2(3ab^2)^3$ **$108a^7b^8$**
5. $(3x^2)(4x^3) + (2x^3)(5x^2)$ **$22x^5$**
6. $(4n^3)n^2 - n^3(3n^2)$ **n^5**
7. $(6p - 2q + 4) + (2p + 3q)$ **$8p + q + 4$**
8. $(3x + y - 2) - (y - x - 3)$ **$4x + 1$**
9. $(4x^3)^2(2x^2y)^3$ **$128x^{12}y^3$**
10. $(7x^5)(2x) + (6x^4)(4x^2)$ **$38x^6$**

NAME ______________________ DATE ____________

4–6 Multiplying Polynomials

Objective: To multiply polynomials.

Example 1 Multiply: $(2x - 3)(x^2 - 4x - 5)$

Solution You can find the product by arranging your work in vertical form. Each term of one polynomial must be multiplied by each term of the other polynomial.

Step 1: Multiply by $2x$.

$$\begin{array}{r} x^2 - 4x - 5 \\ \underline{2x - 3} \\ 2x^3 - 8x^2 - 10x \end{array}$$

Step 2: Multiply by -3.

$$\begin{array}{r} x^2 - 4x - 5 \\ \underline{2x - 3} \\ 2x^3 - 8x^2 - 10x \\ -3x^2 + 12x + 15 \end{array}$$

↑ ↑ Align similar terms.

Step 3: Add the results of Steps 1 and 2.

$$\begin{array}{r} x^2 - 4x - 5 \\ \underline{2x - 3} \\ 2x^3 - 8x^2 - 10x \\ \underline{-3x^2 + 12x + 15} \\ 2x^3 - 11x^2 + 2x + 15 \end{array}$$

Multiply. Use the vertical form.

1. $2a + 1$, $a + 6$ — $2a^2 + 13a + 6$
2. $3n + 6$, $2n - 5$ — $6n^2 - 3n - 30$
3. $3x - 7$, $2x + 1$ — $6x^2 - 11x - 7$
4. $4t - 1$, $3t - 2$ — $12t^2 - 11t + 2$
5. $3x - 4y$, $5x - 2y$ — $15x^2 - 26xy + 8y^2$
6. $2c - 5d$, $3c + d$ — $6c^2 - 13cd - 5d^2$
7. $5c - 3d$, $2c + d$ — $10c^2 - cd - 3d^2$
8. $3x^2 - x - 4$, $x + 4$ — $3x^3 + 11x^2 - 8x - 16$
9. $a^2 - 5a - 7$, $3a + 2$ — $3a^3 - 13a^2 - 31a - 14$
10. $4y^2 - 5y - 2$, $2y - 1$ — $8y^3 - 14y^2 + y + 2$
11. $a^2 - ab + b^2$, $a + b$ — $a^3 + b^3$
12. $2x^2 - xy + y^2$, $2x + y$ — $4x^3 + xy^2 + y^3$

Example 2 Multiply: $(3x - 2)(2x + 5)$

Solution

$(3x - 2)(2x + 5) = (3x - 2)2x + (3x - 2)5$ Use the distributive property.

$= 6x^2 - 4x + 15x - 10$ Combine like terms.

$= 6x^2 + 11x - 10$

Multiply. Use the horizontal form.

13. $(a + 2)(a + 3)$ $a^2 + 5a + 6$
14. $(b + 4)(b + 5)$ $b^2 + 9b + 20$
15. $(x - 3)(x + 8)$ $x^2 + 5x - 24$
16. $(c + 1)(c - 4)$ $c^2 - 3c - 4$
17. $(2a - 1)(a + 4)$ $2a^2 + 7a - 4$
18. $(3a + 4)(a - 1)$ $3a^2 + a - 4$
19. $(2a + 3)(5a - 1)$ $10a^2 + 13a - 3$
20. $(4k - 5)(2k + 6)$ $8k^2 + 14k - 30$
21. $(x - 1)(2x^2 + 3x + 4)$ $2x^3 + x^2 + x - 4$
22. $(2a + 1)(a^2 + 2a + 5)$
23. $(t - 3)(3t^2 + 3t - 4)$
24. $(t - 2)(2t^2 - 3t - 4)$ $2t^3 - 7t^2 + 2t + 8$
25. $(2x - 3)(3x^2 - 4x - 2)$
26. $(3x - 4)(2x^2 - x + 1)$

23. $3t^3 - 6t^2 - 13t + 12$
25. $6x^3 - 17x^2 + 8x + 6$
22. $2a^3 + 5a^2 + 12a + 5$
26. $6x^3 - 11x^2 + 7x - 4$

NAME ______________________ DATE ____________

4–6 Multiplying Polynomials *(continued)*

CAUTION It often is helpful to rearrange the terms of a polynomial so that the degrees of a particular variable are in either increasing order or decreasing order. For example:

In order of decreasing degree of x:

$$x^4 - 2x^3 - 5x + 6$$

In order of increasing degree of x:

$$6 - 5x - 2x^3 + x^4$$

In order of decreasing degree of x and increasing degree of y:

$$x^4 - 5x^3y + 3x^2y^2 - 6xy^3 + 9y^4$$

Example 3 Multiply: $(y + 3x)(x^3 - y^3 + 2x^2y + 3xy^2)$

Solution

$$\begin{array}{l} x^3 - y^3 + 2x^2y + 3xy^2 \\ \underline{y + 3x} \end{array}$$

Rearrange in order of decreasing degree of x and increasing degree of y.

$$\begin{array}{r} x^3 + 2x^2y + 3xy^2 - y^3 \\ \underline{3x + y} \\ 3x^4 + 6x^3y + 9x^2y^2 - 3xy^3 \\ \underline{x^3y + 2x^2y^2 + 3xy^3 - y^4} \\ 3x^4 + 7x^3y + 11x^2y^2 \qquad - y^4 \end{array}$$

Therefore $(y + 3x)(x^3 - y^3 + 2x^2y + 3xy^2) = 3x^4 + 7x^3y + 11x^2y^2 - y^4$.

Multiply using either the horizontal or vertical form. Arrange the terms in each factor in order of decreasing or increasing degree of one of the variables.

30. $-6y^3 - 5y^2 + 19y + 20$

27. $(1 + y)(y^2 + 2y - 3)$ $y^3 + 3y^2 - y - 3$
28. $(4 + x)(x^2 - 4x + 3)$ $x^3 - 13x + 12$
29. $(2 + 3y)(3y - 5 + y^2)$
30. $(3y + 4)(y - 2y^2 + 5)$
31. $(3x + y)(x^2 + 4y^2 + 2xy)$
32. $(1 + 2a)(a^2 - 4 + a)$ $2a^3 + 3a^2 - 7a - 4$
33. $(2x - y)(x^2 + 3y^2 - 4xy)$ $2x^3 - 9x^2y + 10xy^2 - 3y^3$
34. $(y - 3x)(2x^2 + y^2 - 2xy)$ $-6x^3 + 8x^2y - 5xy^2 + y^3$

29. $3y^3 + 11y^2 - 9y - 10$
31. $3x^3 + 7x^2y + 14xy^2 + 4y^3$

Mixed Review Exercises

Solve.

1. $2(x - 1) = 8$ $\{5\}$
2. $3(x - 2) - 2 = 7$ $\{5\}$
3. $4(2a + 3) = 5(a - 6)$ $\{-14\}$

Evaluate if $w = -1$, $x = 2$, and $y = 4$.

4. $x + |w| - y$ -1
5. $w + x + y$ 5
6. $w - |y - x|$ -3
7. $(x + y)^2$ 36
8. $(-x)^2x^2$ 16
9. wy^3 -64

NAME ______________________ DATE ______________

4-7 Transforming Formulas

Objective: To transform a formula.

Example 1 Solve the formula $F = ma$ for m. State the restrictions, if any, for the formula obtained to be meaningful.

Solution

$F = ma$	To get m alone on one side, divide both sides by a.
$\frac{F}{a} = m,\ a \neq 0$	The denominator cannot be 0.

Solve the given formula for the indicated variable. State the restrictions, if any, for the formula obtained to be meaningful.

1. $C = \pi d$ for d **$d = \frac{C}{\pi}$**
2. $F = ma$ for a **$a = \frac{F}{m};\ m \neq 0$**
3. $I = prt$ for t **$t = \frac{I}{pr};\ p \neq 0,\ r \neq 0$**
4. $V = Bh$ for h **$h = \frac{V}{B};\ B \neq 0$**
5. $d = rt$ for t **$t = \frac{d}{r};\ r \neq 0$**
6. $s = gt^2$ for g **$g = \frac{s}{t^2};\ t \neq 0$**

Example 2 The formula $A = \frac{1}{2}h(a + b)$ gives the area of a trapezoid with bases a units and b units and with height h units. Use this formula to solve for the variable b in terms of A, h, and a. State the restrictions, if any, for the formula obtained to be meaningful.

Solution

$A = \frac{1}{2}h(a + b)$	To get clear of fractions, multiply both sides by 2.
$2A = h(a + b)$	Divide both sides by h.
$\frac{2A}{h} = a + b$	Subtract a from both sides.
$\frac{2A}{h} - a = b,\ h \neq 0$	The denominator cannot be 0.

Solve the given formula for the indicated variable. State the restrictions, if any, for the formula obtained to be meaningful.

7. $A = \frac{1}{2}bh$ for h **$h = \frac{2A}{b};\ b \neq 0$**
8. $b = 2b + y$ for y **$y = -b$**
9. $A = \frac{1}{2}h(b + c)$ for h **$h = \frac{2A}{b + c};\ b \neq -c$**
10. $A = P + Prt$ for r **$r = \frac{A - P}{Pt};\ P \neq 0,\ t \neq 0$**
11. $a = 2(l + w)$ for l **$l = \frac{a - 2w}{2}$**
12. $C = \frac{5}{9}(F - 32)$ for F **$F = \frac{9C + 160}{5}$**

33

NAME ______________________ DATE ______________

Example 3 Solve the formula $C = \frac{mv^2}{r}$ for r. State the restrictions, if any, for the formula obtained to be meaningful.

Solution

$C = \frac{mv^2}{r}$	To get r out of the denominator, multiply both sides by r.
$Cr = mv^2$	To get r alone on one side, divide both sides by C.
$r = \frac{mv^2}{C},\ C \neq 0$	The denominator cannot be 0.

Solve the given formula for the indicated variable. State the restrictions, if any, for the formula obtained to be meaningful.

13. $s = \frac{v}{r}$ for v **$v = rs$**
14. $d = \frac{m}{v}$ for m **$m = dv$**
15. $C = \frac{mv^2}{r}$ for m **$\frac{Cr}{v^2} = m,\ v \neq 0$**
16. $2ax + 1 = ax + 5$ for x **$x = \frac{4}{a},\ a \neq 0$**
17. $a = \frac{v - u}{t}$ for u **$u = v - at,\ t \neq 0$**
18. $v^2 = u^2 + 2as$ for a **$a = \frac{v^2 - u^2}{2s},\ s \neq 0$**
19. $S = \frac{n}{2}(a + 1)$ for a **$a = \frac{2S - n}{n},\ n \neq 0$**
20. $m = \frac{x + y + z}{3}$ for x **$x = 3m - y - z$**
21. $l = a + (n - 1)d$ for d **$d = \frac{l - a}{n - 1},\ n \neq 1$**
22. $A = \frac{a + b + c + d}{4}$ for b **$b = 4A - a - c - d$**
23. $3by - 2 = 2by + 1$ for b **$b = \frac{3}{y},\ y \neq 0$**
24. $3aw + 1 = aw - 7$ for a **$a = -\frac{4}{w},\ w \neq 0$**
25. $ax + b = c$ for b **$b = c - ax$**
26. $D = \frac{a}{2}(2t - 1)$ for a **$a = \frac{2D}{2t - 1},\ t \neq \frac{1}{2}$**
27. $am - bm = c$ for a **$a = \frac{bm + c}{m},\ m \neq 0$**
28. $q = 1 + \frac{P}{100}$ for P **$P = 100q - 100$**

Mixed Review Exercises

Simplify.

1. $(y - 4)(y + 2)$ **$y^2 - 2y - 8$**
2. $(2n - 3)(3n - 4)$ **$6n^2 - 17n + 12$**
3. $a[3a - 2(4 + a)]$ **$a^2 - 8a$**
4. $xy(x - 2y)$ **$x^2y - 2xy^2$**
5. $3x(x^2 - 2x + 3)$ **$3x^3 - 6x^2 + 9x$**
6. $(-4x^2)^3$ **$-64x^6$**
7. $n^2 \cdot n^3 \cdot n^4$ **n^9**
8. $(2a^2)^3 \cdot (3a^3b^2)$ **$24a^9b^2$**
9. $(x + 6)(x - 5)$ **$x^2 + x - 30$**
10. $(a + 2b)ab$ **$a^2b + 2ab^2$**
11. $(4m + 5)(8m + 7)$ **$32m^2 + 68m + 35$**
12. $2y^2(y^3 + 2y - 1)$ **$2y^5 + 4y^3 - 2y^2$**

NAME ______________________ DATE ____________

4–8 Rate-Time-Distance Problems

Objective: To solve some word problems involving uniform motion.

Vocabulary **Uniform motion** Motion at a constant speed, or rate.

CAUTION Some problems give the time in *minutes* and the speed in *miles per hour.* Be sure to write the time in terms of hours when you use the given facts.

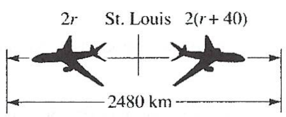
Example 1 **(Motion in opposite directions)** Two jets leave St. Louis at 8:00 A.M., one flying east at a speed 40 km/h greater than the other, which is traveling west. At 10:00 A.M. the planes are 2480 km apart. Find their speeds.

Solution

Step 1 The problem asks for the speeds, or rates, of the planes.

Step 2 Let r = the rate of the plane flying west. Then $r + 40$ = rate of the plane flying east. Make a chart organizing the given facts and use it to label a sketch. Remember that east and west are *opposite directions.*

	Rate	× Time =	Distance
East bound	$r + 40$	2	$2(r + 40)$
West bound	r	2	$2r$



Step 3 The distance between two objects moving in *opposite directions* is the *sum* of the separate distances traveled. The sum of the distances is 2480 km.

$2r + 2(r + 40) = 2480$

Step 4

$2r + 2r + 80 = 2480$
$4r + 80 = 2480$
$4r = 2400$
$r = 600$
$r + 40 = 640$

Step 5 *Check:* In 2 h the eastbound plane flies $2(640) = 1280$ (km). In 2 h the westbound plane flies $2(600) = 1200$ (km). $1280 + 1200 = 2480$ √

The speed of the plane flying west is 600 km/h.
The speed of the plane flying east is 640 km/h.

Solve.

1. Two jets leave Ontario at the same time, one flying east at a speed 20 km/h greater than the other, which is flying west. After 4 h, the planes are 6000 km apart. Find their speeds. **740 km/h, 760 km/h**

2. Two camper vans leave Arrowhead Lake at the same time, one traveling north at a speed of 10 km/h faster than the other, which is traveling south. After 3 h, the camper vans are 420 km apart. Find their speeds. **65 km/h, 75 km/h**

3. Two cars traveled in opposite directions from the same starting point. The rate of one car was 10 km less than the rate of the other. After 4 h the cars were 600 km apart. Find the rate of each car. **70 km/h, 80 km/h**

NAME ______________________ DATE ____________

4–8 Rate-Time-Distance Problems *(continued)*

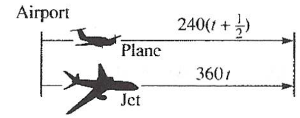
Example 2 **(Motion in the same direction)** A small plane leaves an airport and flies north at 240 mi/h. A jet leaves the airport 30 min later and follows the small plane at 360 mi/h. How long does it take the jet to overtake the small plane?

Solution

Step 1 The problem asks for the jet's flying time before it overtakes the small plane.

Step 2 Let t = the jet's flying time. Then $t + \frac{1}{2}$ = the small plane's flying time. Make a chart organizing the given facts and use it to label a sketch. Notice that 30 min must be written as $\frac{1}{2}$ h.

	Rate ×	Time =	Distance
Plane	240	$t + \frac{1}{2}$	$240(t + \frac{1}{2})$
Jet	360	t	$360t$



Step 3 When the jet overtakes the plane, the distances will be equal.

$360t = 240(t + \frac{1}{2})$

Step 4

$360t = 240t + 120$
$120t = 120$
$t = 1$

Step 5 Check in the words of the problem. The jet overtakes the plane in 1 h.

Solve.

4. A car started out from Memphis toward Little Rock at the rate of 60 km/h. A second car left from the same point 2 h later and drove along the same route at 75 km/h. How long did it take the second car to overtake the first car? **8 h**

5. A tourist bus leaves Richmond at 1:00 P.M. for New York City. Exactly 24 min later, a truck sets out in the same direction. The tourist bus moves at a steady 60 km/h. The truck travels at 80 km/h. How long does it take the truck to overtake the tourist bus? **1 hr 12 min**

6. Exactly 20 min after Alex left home, his sister Alison set out to overtake him. Alex drove at 48 mph and Alison drove at 54 mph. How long did it take Alison to overtake Alex? **2 h 40 min**

7. The McLeans drove from their house to Dayton at 75 km/h. When they returned, the traffic was heavier and they drove at 50 km/h. If it took them 1 h longer to return than to go, how long did it take them to drive home? **3 h**

8. It takes a plane 1 h less to fly from San Diego to New Orleans at 600 km/h than it does to return at 450 km/h. How far apart are the cities? **1800 km**

Mixed Review Exercises

Solve.

1. $32 = -8x$ **{−4}**
2. $3(x + 4) = 27$ **{5}**
3. $(x - 4)(x + 7) = (x + 4)(x - 3)$ **{8}**

4–9 Area Problems

Objective: To solve some problems involving area.

Formula

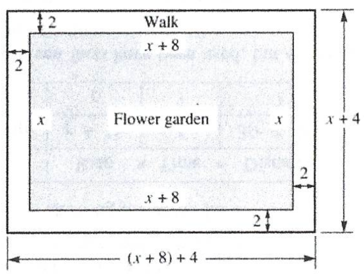
Area of rectangle = length × width

Example A rectangular flower garden is 8 ft longer than it is wide. It is surrounded by a brick walk 2 ft wide. The area of the walk is 176 ft^2. Find the dimensions of the flower garden.

Solution

Step 1 The problem asks for the dimensions of the flower garden. Make a sketch.

Step 2 Let x = the width of the flower garden. Then $x + 8$ = the length of the flower garden. Label your sketch.



Step 3 Area of walk = Area of garden and walk − Area of garden

$$176 = (x + 12)(x + 4) - x(x + 8)$$

Step 4

$$176 = x^2 + 16x + 48 - x^2 - 8x$$
$$176 = 8x + 48$$
$$128 = 8x$$
$$16 = x \text{ and } x + 8 = 24$$

Step 5 *Check:* If the dimensions of the flower garden are 16 ft and 24 ft, the dimensions of the flower garden and walk are 20 ft and 28 ft.

Area of the flower garden and walk = $20 \cdot 28 = 560\ (ft^2)$
Area of the flower garden = $16 \cdot 24 = 384\ (ft^2)$
Area of walk = $560 - 384 = 176\ (ft^2)$

The dimensions of the flower garden are 24 ft and 16 ft.

4–9 Area Problems *(continued)*

Solve.

1. A rectangle is twice as long as it is wide. If its length and width are both decreased by 4 cm, its area is decreased by 164 cm^2. Find its original dimensions. **15 cm by 30 cm**
2. A rectangle is three times as long as it is wide. If its length and width are both increased by 3 m, its area is increased by 81 m^2. Find its original dimensions. **6 m by 18 m**
3. A rectangle is 8 cm longer than it is wide. If its length and width are both increased by 2 cm, its area is increased by 68 cm^2. Find its original dimensions. **12 cm by 20 cm**
4. A rectangle is 12 cm longer than it is wide. If its length and width are both decreased by 2 cm, its area is decreased by 108 cm^2. Find its original dimensions. **22 cm by 34 cm**
5. A rectangular fish pond is 8 ft longer than it is wide. A wooden walk 2 ft wide is placed around the pond. The area covered by the pond and walk is 160 ft^2 greater than the area covered by the pond alone. What are the dimensions of the pond? **22 ft by 14 ft**
6. A rectangular swimming pool is 10 m longer than it is wide. A walkway 2 m wide surrounds the pool. Find the dimensions of the pool if the area of the walkway is 216 m^2. **20 m by 30 m**
7. A poster is 24 cm taller than it is wide. If it is mounted on a piece of cardboard so that there is a 6 cm border on all sides and if the area of the border alone is 720 cm^2, what are the dimensions of the poster? **12 cm by 36 cm**
8. A poster is 20 cm longer than it is wide. It is mounted on a piece of cardboard so that there is a 10 cm border on all sides. If the area of the border is 2400 cm^2, what is the area of the poster? **2400 cm^2**
9. A house has two rooms of equal area. One room is square and the other is 6 ft narrower and 9 ft longer than the square room. Find the area of each room. **324 ft^2; 324 ft^2**
10. A baker has two pans with the same area. One pan is square and the other is 6 cm narrower and 8 cm longer than the square pan. Find the area of each pan. **576 cm^2; 576 cm^2**

Mixed Review Exercises

Simplify.

1. $(-3 + 7) + (-6)$ **−2**
2. $4(c - 1) + (2)c + 7$ **6c + 3**
3. $(3x^2y)^4$ **$81x^8y^4$**
4. $y - (-5) - [y + (-3)]$ **8**
5. $4^3 \cdot 2^2$ **256**
6. $(5 \cdot 4 + 3 \cdot 4) \div (4 \cdot 2)$ **4**
7. $-\frac{8}{3} + \frac{2}{3} + 2$ **0**
8. $\left(\frac{1}{3}x^3\right)\left(\frac{3}{5}x^2\right)$ **$\frac{1}{5}x^5$**
9. $-\frac{x}{8}(-96)$ **12x**
10. $(-7 + 11) + (-4)$ **0**
11. $\left(\frac{1}{2}x^2\right)\left(\frac{2}{5}x\right)$ **$\frac{1}{5}x^3$**
12. $(2x^2y)^3$ **$8x^6y^3$**

NAME ________________ DATE ________

4–10 Problems Without Solutions

Objective: To recognize problems that do not have solutions.

CAUTION Not all word problems have solutions. Here are some reasons for this:

1. Not enough information is given.
2. The given facts lead to an unrealistic result. (The result satisfies the equation used, but not the conditions of the problem situation.)
3. The given facts are contradictory. (They cannot all be true at the same time.)

Example 1 On a trip of 210 km, Roberto went by train for 3h and by bus for the rest of the trip. The average speed of the train was 15 km/h more than that of the bus. Find the average speed of the bus.

Solution

Step 1 The problem asks for the speed of the bus.

Step 2 Let r = the speed of the bus.
Then $r + 15$ = the speed of the train.

Make a chart showing the given facts.

	Rate	× Time =	Distance
Train trip	$r + 15$	3	$3(r + 15)$
Bus trip	r	?	?

All of the given facts have been used, but *not enough information has been given to write an equation.*

The problem does not have a solution.

Example 2 Angela says she has the same number of nickels as dimes. Three times the value of the dimes is equal to 20¢ more than six times the value of the nickels. How many of each kind does she have?

Solution

Step 1 The problem asks for the number of nickels and dimes.

Step 2 Let x = the number of nickels and let x = the number of dimes.
Then $5x$ = the value (¢) of the nickels and $10x$ = the value (¢) of the dimes.

Step 3 $3(10x) = 20 + 6(5x)$

Step 4 $30x = 20 + 30x$
$0 = 20$

The false statement "0 = 20" tells you the *given facts are contradictory.*

The problem has no solution.

NAME ________________ DATE ________

4–10 Problems Without Solutions *(continued)*

Example 3 A bank teller cashes a check for \$100 and gives the customer the same number of \$10 bills and \$20 bills. How many of each type of bill will the customer get?

Solution

Step 1 The problem asks for the number of bills of each type the customer will get.

Step 2 Let x = the number of each kind of bill.

Step 3 $10x + 20x = 100$

Step 4 $30x = 100$
$x = 3\frac{1}{3}$
Since you can't have $\frac{1}{3}$ of a bill, the given facts lead to an *unrealistic result.* The problem does not have a solution.

Solve each problem that has a solution. If a problem has no solution, explain why.

1. You plan a lawn 10 ft longer than it is wide. It is to be surrounded by a flower bed 3 ft wide. Find the dimensions of the lawn if the flower bed covers 80 ft^2. **No solution; unrealistic result**
2. A bank teller was asked to cash a check for \$180 and to give the customer the same number of \$10 bills and \$20 bills. How many bills of each kind did the teller count out? **six \$10 bills, six \$20 bills**
3. In the course of a year, the sum of an investor's gains and losses was \$2400. What were his gains that year? **No solution; not enough information given**
4. Find three consecutive integers whose sum is 47 more than the largest integer. **23, 24, 25**
5. Find two consecutive integers whose sum is 48. **No solution; given facts contradictory**
6. Chan bought more 25¢ stamps than 15¢ stamps. How many of each kind did he buy if the total cost of the stamps is \$7.80? **No solution; not enough information given**
7. Hans has as many dimes as quarters. The dimes and quarters total \$10. How many of each kind does he have? **No solution; unrealistic result**
8. Last month Charlene worked as many 8 h shifts as she worked 10 h shifts. How many 8 h shifts did she work last month? **No solution; not enough information given**
9. Georgette has the same number of 25¢ stamps as she does 15¢ stamps. Six times the value of the 25¢ stamps is a dollar more than 10 times the value of the 15¢ stamps. How many of each type of stamp does she have? **No solution; given facts contradictory**
10. Last month Theo worked the same number of hours on 8 h shifts as he did on 6 h shifts. One fourth the amount of time he put in at 8 h shifts was 1 h less than one third the time he put in at 6 h shifts. How many hours did he work at each type of shift? **No solution; given facts contradictory**

Mixed Review Exercises

Solve.

1. $-5x = 40$ **{−8}**
2. $2(n - 3) = 28$ **{17}**
3. $32 = -4x$ **{−8}**
4. $-n + 6 = 2$ **{4}**
5. $x - 5 = |3 - 7|$ **{9}**
6. $-y + 15 = 10$ **{5}**
7. $-\frac{1}{2}(x + 4) = 3$ **{−10}**
8. $\frac{1}{5}x = 12$ **{60}**