

## 5-2 Dividing Monomials

**Objective:** To simplify quotients of monomials and to find the GCF of several monomials.

### Vocabulary

**Greatest common factor (GCF) of two or more monomials** The common factor with the *greatest coefficient* and the *greatest degree* in each variable. For example,  $5x^2y$  is the GCF of  $10x^2y^2$  and  $25x^3y$ .

### Properties and Rules

#### Property of Quotients

If  $a$ ,  $b$ ,  $c$ , and  $d$  are real numbers with  $b \neq 0$ , and  $d \neq 0$ , then  $\frac{ac}{bd} = \frac{a}{b} \cdot \frac{c}{d}$ .

For example,  $\frac{15}{16} = \frac{3 \cdot 5}{2 \cdot 8} = \frac{3}{2} \cdot \frac{5}{8}$

#### Rule for Simplifying Fractions

If  $b$ ,  $c$ , and  $d$  are real numbers with  $b \neq 0$  and  $d \neq 0$ , then  $\frac{bc}{bd} = \frac{c}{d}$ .

For example,  $\frac{15}{18} = \frac{3 \cdot 5}{3 \cdot 6} = \frac{5}{6}$ .

#### Rule of Exponents for Division

If  $a$  is a nonzero real number and  $m$  and  $n$  are positive integers, then:

$$\begin{array}{lll} \text{If } m > n: & \text{If } n > m: & \text{If } m = n: \\ \frac{a^m}{a^n} = a^{m-n} & \frac{a^m}{a^n} = \frac{1}{a^{n-m}} & \frac{a^m}{a^n} = 1 \end{array}$$

**CAUTION** You can divide the numerator and denominator of a fraction only by a nonzero number. In the examples of this lesson, *assume that no denominator equals zero.*

**Example 1** Simplify: a.  $\frac{28}{35}$  b.  $\frac{-15xy}{21x}$

**Solution** a. Divide both numerator and denominator by 7. The “cancel” marks show this.

$$\frac{28}{35} = \frac{4 \cdot \cancel{7}}{5 \cdot \cancel{7}} = \frac{4}{5}$$

b. Divide both numerator and denominator by  $3x$ .

$$\frac{-15xy}{21x} = \frac{\cancel{3x}(-5y)}{\cancel{3x} \cdot 7} = \frac{-5y}{7}, \text{ or } -\frac{5y}{7}$$

**Example 2** Simplify: a.  $\frac{x^8}{x^3}$  b.  $\frac{x^3}{x^8}$  c.  $\frac{x^2}{x^2}$

**Solution** a.  $\frac{x^8}{x^3} = x^{8-3} = x^5$  b.  $\frac{x^3}{x^8} = \frac{1}{x^{8-3}} = \frac{1}{x^5}$  c.  $\frac{x^2}{x^2} = 1$

**5-2 Dividing Monomials** (continued)

Simplify. Assume that no denominator equals zero.

1.  $\frac{25}{30}$       2.  $\frac{48}{72}$       3.  $\frac{54}{72}$       4.  $\frac{10^3}{10^6}$       5.  $\frac{10^8}{10^5}$       6.  $\frac{10a}{2a}$
7.  $\frac{12m}{4m}$       8.  $\frac{15 \cdot 10^3}{5 \cdot 10^4}$       9.  $\frac{6x^4}{9x^2}$       10.  $\frac{4n^6}{20n^4}$       11.  $\frac{2x^5}{16x^4}$       12.  $\frac{12y^3}{3xy^2}$
13.  $\frac{4a^2b}{16ab^2}$       14.  $\frac{-6x^2y^3}{9xy^2}$       15.  $\frac{-8a^2b}{-20ab}$       16.  $\frac{-32cd^3}{-24bd^2}$       17.  $\frac{-21bc^3}{-14cd^2}$
18.  $\frac{30xz^3}{-35yz^2}$       19.  $\frac{x^2yz^3}{x^3y^3z^3}$       20.  $\frac{a^2b^4c}{a^2bc^3}$       21.  $\frac{35a^2b^3c}{25abc}$       22.  $\frac{26x^2yz}{52xyz}$

**Example 3**  $\frac{(9x)^2}{(3x)^3} = \frac{81x^2}{27x^3} = \frac{\cancel{27}x^2 \cdot 3}{\cancel{27}x^2 \cdot x} = \frac{3}{x}$

Simplify. Assume that no denominator equals zero.

23.  $\frac{(2x)^3}{2x^3}$       24.  $\frac{5m^2}{(5m)^2}$       25.  $\frac{(2t^2)^3}{(2t^3)^2}$       26.  $\frac{(4a^2)^3}{(4a^3)^2}$       27.  $\frac{(3ab)^2}{3a^2b}$
28.  $\frac{(2mn)^3}{2m^3n^2}$       29.  $\frac{(-z)^6}{(-z)^3}$       30.  $\frac{(-a)^5}{(-a)^3}$       31.  $\frac{(-xy)^7}{xy^7}$       32.  $\frac{(-t^3)^4}{(-t^2)^5}$

**Example 4** Find the missing factor.  $45x^2y^3z^4 = (3xyz^2)(?)$       **Solution**  $\frac{45x^2y^3z^4}{3xyz^2} = 15xy^2z^2$

Find the missing factor.

33.  $8t^4 = (2t)(?)$       34.  $10w^4 = (2w^2)(?)$       35.  $6a^3b^5 = (2a^2b^2)(?)$
36.  $15pq^3 = (5pq)(?)$       37.  $-28x^2y^4 = (7x^2y)(?)$       38.  $-32a^5b^4 = (-8a)(?)$

**Example 5** Find the GCF of  $18x^3y$  and  $10x^2y^3$ .

**Solution**  $\left. \begin{array}{l} 18 = 2 \cdot 3^2 \\ 10 = 2 \cdot 5 \end{array} \right\} \text{GCF} = 2$        $\left. \begin{array}{l} x^3y \\ x^2y^3 \end{array} \right\} \text{GCF} = x^2y$        $\left. \begin{array}{l} \text{GCF} = 2 \\ \text{GCF} = x^2y \end{array} \right\} \text{GCF} = 2x^2y$

Find the GCF.

39.  $21x^3, 14x^2$   
 40.  $a^3b^2, a^2b^3$   
 41.  $6xy^2, 8x^4y^3$   
 42.  $18c^2d^3, 24c^2d$   
 43.  $35pq^2r, 25p^3qr^2$

**Mixed Review Exercises**

Simplify.

1.  $\frac{1}{4}(-24)$       2.  $105 \cdot \frac{1}{5}$       3.  $378 \div 9$       4.  $4n^3 \left( \frac{1}{4}n^3 \right)$       5.  $12 \div \left( -\frac{1}{3} \right)$       6.  $10y \cdot \frac{2}{5}y^2$