

Section 2.3 – Calculating Limits Using Limit Laws

Let a and c denote constants. Suppose the limits

$$\lim_{x \rightarrow a} f(x) \quad \text{and} \quad \lim_{x \rightarrow a} g(x)$$

exist. Then

1. Sum Law:

$$\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

2. Difference Law:

$$\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$$

3. Constant Multiple Law:

$$\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x)$$

4. Product Law:

$$\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

5. Quotient Law:

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \quad \text{if } \lim_{x \rightarrow a} g(x) \neq 0$$

6. Constant Function Law:

$$\lim_{x \rightarrow a} c = c$$

7. Trivial Law:

$$\lim_{x \rightarrow a} x = a$$

8. Power Law:

$$\lim_{x \rightarrow a} [f(x)]^n = \left[\lim_{x \rightarrow a} f(x) \right]^n \quad \text{when } n \text{ is a positive integer}$$

In particular, $\lim_{x \rightarrow a} x^n = a^n$.

9. Root Law:

$$\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \begin{cases} \sqrt[n]{\lim_{x \rightarrow a} f(x)} & \text{if } n \text{ is odd} \\ \sqrt[n]{\lim_{x \rightarrow a} f(x)} & \text{if } n \text{ is even and } \lim_{x \rightarrow a} f(x) > 0 \\ \text{does not exist} & \text{if } n \text{ is even and } \lim_{x \rightarrow a} f(x) < 0 \end{cases}$$

10. Direct Substitution Property for Polynomials: If $p(x)$ denotes a polynomial, then

$$\lim_{x \rightarrow a} p(x) = p(a).$$

11. Direct Substitution Property for Rational Functions: If $r(x)$ denotes a rational function, then

$$\lim_{x \rightarrow a} r(x) = r(a).$$

if a belongs to the domain of r .

12. Limits of functions that are equal except at a point: If $g(x) = f(x)$ when $x \neq a$ and if $\lim_{x \rightarrow a} f(x)$ exists, then

$$\lim_{x \rightarrow a} g(x) = \lim_{x \rightarrow a} f(x).$$