

6-3 Dividing Fractions

Objective: To divide algebraic fractions.

Division Rule for Fractions To divide by a fraction, you multiply by its reciprocal. Remember that the reciprocal of a number n is the number $\frac{1}{n}$ for which $n \cdot \frac{1}{n} = 1$.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \quad \text{For example, } \frac{3}{5} \div \frac{2}{7} = \frac{3}{5} \cdot \frac{7}{2} = \frac{21}{10}.$$

Example 1 Divide: $\frac{x}{3y} \div \frac{xy}{9}$

Solution $\frac{x}{3y} \div \frac{xy}{9} = \frac{x}{3y} \cdot \frac{9}{xy}$ Multiply by the reciprocal of $\frac{xy}{9}$.

$$= \frac{\cancel{x} \cdot \cancel{3} \cdot 3}{\cancel{3} \cdot y \cdot \cancel{x} \cdot y}$$
 Factor and simplify.

$$= \frac{3}{y^2}$$

Divide. Give your answers in simplest form.

1. $\frac{8}{5} \div \frac{16}{25}$

2. $\frac{3}{4} \div \frac{9}{8}$

3. $\frac{a}{10} \div \frac{a}{2}$

4. $\frac{2x}{5} \div \frac{x}{15}$

5. $\frac{x^2}{y} \div \frac{x}{y^2}$

6. $\frac{4n^2}{5} \div \frac{8n}{25}$

7. $\frac{ab}{4} \div \frac{a}{b}$

8. $\frac{c}{3d} \div \frac{c^2}{9d^2}$

9. $\frac{2x^2}{3y} \div \frac{xy}{9}$

10. $\frac{3n}{4m^2} \div \frac{1}{12mn}$

11. $\frac{x^2y}{2} \div xy$

12. $\frac{8a^2}{3b} \div 4a$

13. $1 \div \left(\frac{2x}{3}\right)^2$

14. $9 \div \left(\frac{3}{n}\right)^2$

15. $16 \div \left(\frac{2}{a}\right)^3$

Example 2 Divide: $\frac{15}{x^2 - 16} \div \frac{20}{x - 4}$

Solution $\frac{15}{x^2 - 16} \div \frac{20}{x - 4} = \frac{15}{x^2 - 16} \cdot \frac{x - 4}{20}$ Multiply by the reciprocal.

$$= \frac{\cancel{5} \cdot 3}{(x + 4)(\cancel{x} - 4)} \cdot \frac{\cancel{x} - 4}{\cancel{5} \cdot 4}$$
 Factor and simplify.

$$= \frac{3}{4(x + 4)}$$

Divide. Give your answers in simplest form.

16. $\frac{3 + 3b}{6} \div \frac{1 + b}{9}$

17. $\frac{4n - 2}{8n} \div \frac{2n - 1}{24}$

18. $\frac{x^2 - 4}{3} \div \frac{x + 2}{9}$

19. $\frac{x^2 - 16}{3x} \div \frac{x - 4}{6}$

20. $\frac{x^2 - 9}{3} \div \frac{x + 3}{6}$

21. $\frac{x^2 - 25}{4x} \div \frac{x - 5}{12}$

6-3 Dividing Fractions (continued)

Divide. Give your answers in simplest form.

22. $\frac{2}{x-3} \div \frac{2}{3-x}$

23. $\frac{4}{6-3a} \div \frac{6}{8-4a}$

24. $\frac{x^2+2x}{x^2-4} \div \frac{x+2}{x-2}$

25. $\frac{1}{3a-12} \div \frac{1}{2a-8}$

26. $\frac{3x-3y}{x} \div \frac{x^2-y^2}{x^2}$

27. $\frac{4}{n^2-16} \div \frac{8n-32}{n+4}$

Example 3 Divide: $\frac{x^2-3x-10}{2x-6} \div \frac{x^2-4}{x^2+x-6}$

Solution

$$\begin{aligned} \frac{x^2-3x-10}{2x-6} \div \frac{x^2-4}{x^2+x-6} &= \frac{x^2-3x-10}{2x-6} \cdot \frac{x^2+x-6}{x^2-4} \\ &= \frac{(x-5)\cancel{(x+2)}}{2(x-3)} \cdot \frac{(x+3)\cancel{(x-2)}}{\cancel{(x+2)}\cancel{(x-2)}} \\ &= \frac{(x-5)(x+3)}{2(x-3)} \end{aligned}$$

{ Stop; no further simplification is possible.

Divide. Give your answers in simplest form.

28. $\frac{x^2-9}{x^2-4} \div \frac{x^2-x-6}{x^2+x-6}$

29. $\frac{x^2+x-20}{5x+25} \div \frac{x^2-4x-5}{x^2-25}$

30. $\frac{x^2-y^2}{x^2+y^2} \div (x-y)$

31. $\frac{x^2-x-6}{x^2+2x+1} \div \frac{x+2}{x+1}$

32. $\frac{x^2-3x+2}{x^2+3x+2} \div \frac{4x-8}{8x+8}$

33. $\frac{x^2-4}{x+2} \div \frac{x-2}{x+1}$

34. $\frac{x^2-25}{x^2-16} \div \frac{4x+20}{8x-32}$

35. $\frac{4x^2-y^2}{4y^2-x^2} \div \frac{2x-y}{2y-x}$

36. $\frac{x^2-3x+2}{x^2-7x+10} \div \frac{x^2-1}{x^2-4x-5}$

37. $\frac{x^2-8x+15}{x^2-9x+14} \div \frac{x^2-9}{x^2+x-6}$

38. $\frac{2x^2+7x+3}{2x^2+5x+2} \div \frac{x^2-7x-30}{x^2-6x-40}$

39. $\frac{x^2+5x-6}{x^2-x-20} \div \frac{x^2+2x-3}{x^2-2x-15}$

Mixed Review Exercises

Solve.

1. $3k = 4k - 11$

2. $5p + 10 = 45$

3. $(4b - 3) - (3 - 2b) = 30$

4. $\frac{1}{3}(9k - 6) = 7$

5. $2n^3 - 32n = 0$

6. $2x^2 + x = 3$

Give the prime factorization of each number.

7. 225

8. 136

9. 140

10. 1250