Math 122 - Quiz 7 Name Key

February 11, 2019

1. $\int u^{-1}\,du =$ $\ln|u| + C$. (2 pts.)

2. $\int \sec(u)\tan(u)\,du =$ $\sec u + C$. (2 pts.)

3. Find the following indefinite integral: (2 pts.)

$$\int \frac{1-\sqrt{x}}{x}\,\cancel{dx}\,dx$$

$$\int \frac{1-\sqrt{x}}{x}\,dx = \int\left(\frac{1}{x} - \frac{\sqrt{x}}{x}\right)dx = \int \frac{dx}{x} - \int x^{-1/2}\,dx$$

$$= \ln|x| - 2\sqrt{x} + C$$

4. Verify by differentiation that (2 pts.)

$$\int \tan^2 x\,dx = \tan x - x + C.$$

$$\frac{d}{dx}\left(\tan x - x + C\right) = \sec^2 x - 1.$$

Use the trig identity $\tan^2 x + 1 = \sec^2 x$.

And so $\frac{d}{dx}\left(\tan x - x + C\right) = \tan^2 x$, which is the integrand!

Proof:

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\Rightarrow \tan^2 x + 1 = \sec^2 x$$

5. Find the value of the sum: (2 pts.)

$$\sum_{i=0}^{2}(2^i + i^2).$$

$$\sum_{i=0}^{2}(2^i + i^2) = (2^0 + 0^2) + (2^1 + 1^2) + (2^2 + 2^2)$$

$$= 1 + 3 + 8 = \boxed{12}$$

Extra Credit. (1 pt.) Find the value of $\sum_{i=1}^{36} i^2$.

$$\sum_{i=1}^{36} i^2 = \left.\frac{n(n+1)(2n+1)}{6}\right|_{n=36} = \frac{(36)(37)(73)}{6} = 6(37)(73)$$

$$= 16,206.$$