

NET CHANGE THEOREM

The FTC-2 can be rewritten as

$$\int_a^b F'(x) dx = F(b) - F(a).$$

$F'(x)$ denotes the *rate of change of the function F at x* . The difference $F(b) - F(a)$ is the *net change in the function F when x changes from a to b* . In words, we have

Theorem (Net Change Theorem). *The net change in a function from a to b is equal to the integral of the rate of change of the function over the interval $[a, b]$.*

Example 1. A bacteria population is 4,000 at time $t = 0$ and its rate of growth is $1000 \cdot 2^t$ bacteria per hour after t hours. What is the population after one hour. (Stewart #71, p. 411)

Suppose a particle moves along a straight line and that its position at time t is given by a function $s(t)$. Since its velocity is $v(t) = s'(t)$, we have from the FTC-2 that

$$\int_{t_1}^{t_2} v(t) dt = s(t_2) - s(t_1).$$

Example 2. The velocity (in meters per second) of a particle moving along a straight line is $v(t) = 3t - 5$. Find (a) the displacement (net change in position) and (b) the distance traveled by the particle during the time interval $0 \leq t \leq 3$. (Stewart #59, p. 410)