$\boxed{\text{Quiz 19}}$

$\boxed{\#23\ p.508}$ Evaluate $\int_0^1 (1+\sqrt{x})^8\,dx$.

Soln. Let $u = 1+\sqrt{x}$. Then $x = (u-1)^2$ & $dx = 2(u-1)\,du$.

Hence, $\int_0^1 (1+\sqrt{x})^8\,dx = \int_1^2 u^8 \cdot 2(u-1)\,du = 2\int_1^2 (u^9 - u^8)\,du$

$$= \left[\frac{u^{10}}{5} - \frac{2}{9}u^9\right]_1^2 = \left(\frac{1024}{5} - \frac{1024}{9}\right) - \left(\frac{1}{5} - \frac{2}{9}\right)$$

$$= 1024\left(\frac{1}{5} - \frac{1}{9}\right) - \left(\frac{9-10}{45}\right) = 1024\left(\frac{4}{45}\right) + \frac{1}{45}$$

$$= \boxed{\frac{4097}{45}}$$

$\boxed{\#26\ p\,508}$ Evaluate $\int_0^1 \frac{3x^2+1}{x^3+x^2+x+1}\,dx$.

Soln. $x^3+x^2+x+1 = x^2(x+1) + (x+1) = (x+1)(x^2+1)$

$\frac{3x^2+1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} \Rightarrow A(x^2+1) + (Bx+C)(x+1) = 3x^2+1.$

So, $2(x^2+1) + (Bx+C)(x+1) = 3x^2+1$

Let $x=-1$: $2A = 4 \Rightarrow \underline{A=2}$.

Let $x=0$: $2 + C = 1 \Rightarrow \underline{C=-1}$. Thus, we now have

$2(x^2+1) + (Bx-1)(x+1) = 3x^2+1$.

Let $x=1$: $4 + (B-1)(2) = 4 \Rightarrow 2(B-1) = 0 \Rightarrow \underline{B=1}$

Hence,

$$\int_0^1 \frac{3x^2+1}{x^3+x^2+x+1}\,dx = \int_0^1 \frac{2}{x+1}\,dx + \int_0^1 \frac{x\,dx}{x^2+1} - \int_0^1 \frac{1}{x^2+1}\,dx$$

$$= 2\ln(x+1)\Big|_0^1 - \frac{1}{2}\ln(x^2+1)\Big|_0^1 - \tan^{-1}x\Big|_0^1$$

$$= 2\ln 2 + \frac{1}{2}\ln 2 - \frac{\pi}{4}$$

$$= \frac{5}{2}\ln 2 - \frac{\pi}{4}$$

36 p508 Evaluate $\int \frac{1+\sin x}{1+\cos x}\,dx = I.$

Soln. $\frac{1+\sin x}{1+\cos x} = \frac{1+\sin x}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x} = \frac{1+\sin x-\cos x-\sin x\cos x}{\sin^2 x}$

$= \csc^2 x + \csc x - \cot x\csc x - \frac{\cos x}{\sin x}.$

$\therefore\ I = \int \csc^2 x\,dx + \int \csc x\,dx - \int \csc x\cot x\,dx - \int \frac{\cos x\,dx}{\sin x}$

$= -\cot x - \ln|\csc x + \cot x| + \csc x - \ln|\sin x| + C$

Or, $I = -\cot x + \ln|\csc x - \cot x| + \csc x - \ln|\sin x| + C$ (See p 3 for another form)

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44 p508 Evaluate $\int \sqrt{1+e^x}\,dx.$

Soln. Let $u = \sqrt{1+e^x}$. Then $u^2 = 1+e^x$. So, $2u\frac{du}{dx} = e^x$

$\Rightarrow 2u\,du = (u^2-1)\,dx \Rightarrow dx = \frac{2u}{u^2-1}\,du.$

Thus, $\int\sqrt{1+e^x}\,dx = \int u\cdot\frac{2u}{u^2-1}\,du = \int\frac{2u^2}{u^2-1}\,du$

$= 2\int\frac{u^2-1+1}{u^2-1}\,du = 2\int\left(1+\frac{1}{u^2-1}\right)du.$

$\frac{1}{u^2-1} = \frac{1}{(u-1)(u+1)} = \frac{A}{u-1} + \frac{B}{u+1}$

$\Rightarrow A(u+1) + B(u-1) = 1.$

Let $u = 1$: $2A = 1 \Rightarrow A = \frac{1}{2}$. Let $u = -1$. Then $-2B = 1 \Rightarrow B = -\frac{1}{2}.$

Hence,

$\int\sqrt{1+e^x}\,dx = 2\int\left(1 + \frac{1}{2}\cdot\frac{1}{u-1} - \frac{1}{2}\cdot\frac{1}{u+1}\right)du$

$= 2u + \int\frac{du}{u-1} - \int\frac{du}{u+1}$

$= 2u + \ln|u-1| - \ln|u+1| + C$

and so

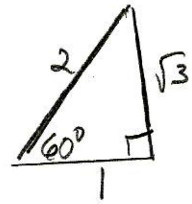
$\int\sqrt{1+e^x}\,dx = 2\sqrt{1+e^x} + \ln\left(\sqrt{1+e^x}-1\right) - \ln\left(\sqrt{1+e^x}+1\right) + C$

$\boxed{66\ p.508}$ Evaluate $\displaystyle\int_{\pi/4}^{\pi/3} \frac{\ln(\tan x)}{\sin x \cos x}\,dx =: I$.

Soln. $\displaystyle\int \frac{\ln(\tan x)}{\sin x \cos x}\,dx = \int \frac{\ln(\tan x)}{\left(\frac{\sin x}{\cos x}\right)\cos^2 x}\,dx = \int \frac{\ln(\tan x)}{\tan x}\sec^2 x\,dx.$

Let $u = \tan x$. Then $du = \sec^2 x\,dx$.

So, $\displaystyle I = \int_{\tan(\pi/4)}^{\tan(\pi/3)} \frac{\ln u}{u}\,du = \int_1^{\sqrt{3}} \frac{\ln u}{u}\,du.$



$\tan 60^\circ = \sqrt{3}$

Let $v = \ln u$. Then $dv = \frac{1}{u}\,du$. So,

$$I = \int_0^{\ln\sqrt{3}} v\,dv = \frac{1}{2}\left[v^2\right]_0^{\ln\sqrt{3}} = \frac{1}{2}\left(\ln\sqrt{3}\right)^2 = \frac{1}{2}\left(\frac{1}{2}\ln 3\right)^2 = \frac{1}{8}(\ln 3)^2$$

$\boxed{\#36\ p508}$ Another soln is

$$\int \frac{1+\sin x}{1+\cos x}\,dx = \int \frac{1}{1+\cos x}\,dx + \int \frac{\sin x}{1+\cos x}\,dx.$$

$$\int \frac{\sin x}{1+\cos x}\,dx = \int \frac{-du}{u} = -\ln|1+\cos x| + K_1 \quad \text{letting } u = 1+\cos x.$$

also,

$$\int \frac{1}{1+\cos x}\,dx = \int \frac{1-\cos x}{1-\cos^2 x}\,dx = \int \frac{1-\cos x}{\sin^2 x}\,dx = \int \frac{1}{\sin^2 x}\,dx - \int \frac{\cos x}{\sin^2 x}\,dx$$

$$= \int \csc^2 x\,dx - \int \frac{1}{u^2}\,du \quad (u = \sin x)$$

$$= -\cot x + \frac{1}{u} + K_2 = -\frac{\cos x}{\sin x} + \frac{1}{\sin x} + K_2$$

$$= \frac{1-\cos x}{\sin x} + K_2$$

$$\therefore \int \frac{1+\sin x}{1+\cos x}\,dx = \frac{1-\cos x}{\sin x} - \ln|1+\cos x| + C$$