NAME _____ DATE _____

12–4 Graphs of Quadratic Equations: The Discriminant

Objective: To use the discriminant to find the number of roots of the equation $ax^2 + bx + c = 0$ and the number of x-intercepts of the graph of the related equation $y = ax^2 + bx + c$.

Vocabulary

x-intercept The x-coordinate of a point where the curve of a parabola intersects the x-axis.

Discriminant For a quadratic equation in the form $ax^2 + bx + c = 0$, the value of $b^2 - 4ac$ is called the discriminant.

	Value of $b^2 - 4ac$	Number of different real roots of $ax^2 + bx + c = 0$	Number of x-intercepts of the graph of $y = ax^2 + bx + c$
Case 1	positive	2	2
Case 2	zero	1 (a double root)	1
Case 3	negative	0	0

Example 1 Write the value of the discriminant of each equation. Then use it to decide how many different real-number roots the equation has. (Do not solve.)

a. $x^2 - 4x - 5 = 0$ **b.** $x^2 - 6x + 9 = 0$ **c.** $x^2 - 3x + 5 = 0$

Solution **a.** $x^2 - 4x - 5 = 0$

Substitute $a = 1$, $b = -4$, and $c = -5$ in the discriminant formula.

$b^2 - 4ac = (-4)^2 - 4(1)(-5) = 16 - 4(-5) = 16 + 20 = 36$

Since the discriminant is positive, $x^2 - 4x - 5 = 0$ has two real roots.

b. $x^2 - 6x + 9 = 0$

Substitute $a = 1$, $b = -6$, and $c = 9$ in the discriminant formula.

$b^2 - 4ac = (-6)^2 - 4(1)(9) = 36 - 36 = 0$

Since the discriminant is zero, $x^2 - 6x + 9 = 0$ has one real root.

c. $x^2 - 3x + 5 = 0$

Substitute $a = 1$, $b = -3$, and $c = 5$ in the discriminant formula.

$b^2 - 4ac = (-3)^2 - 4(1)(5) = 9 - 20 = -11$

Since the discriminant is negative, $x^2 - 3x + 5 = 0$ has no real roots.

NAME _____ DATE _____

12–4 Graphs of Quadratic Equations: The Discriminant *(continued)*

Write the value of the discriminant of each equation. Then use it to decide how many different real-number roots the equation has. (Do not solve.)

1. $x^2 - 3x + 2 = 0$
2. $x^2 - 3x + 5 = 0$
3. $x^2 - 8x + 16 = 0$
4. $2x^2 - 5x - 4 = 0$
5. $4y^2 - 12y + 9 = 0$
6. $3t^2 - 5t + 3 = 0$
7. $2n^2 + n - 6 = 0$
8. $5y^2 - 8y + 4 = 0$
9. $2x^2 - 7x + 5 = 0$
10. $4m^2 - 20m + 25 = 0$
11. $3x^2 - 7x + 2 = 0$
12. $-3b^2 + 2b - 3 = 0$
13. $-2x^2 + 6x - 3 = 0$
14. $3x^2 - 4x = 6$
15. $x^2 - x + 1 = 0$

Example 2 Determine (a) how many x-intercepts the parabola $y = 4x - x^2 + 5$ has and (b) whether its vertex lies above, below, or on the x-axis. (Do not draw the graph.)

Solution

a. The x-intercepts of the graph are the roots of the equation

$$0 = 4x - x^2 + 5, \quad \text{or} \quad -x^2 + 4x + 5 = 0.$$

Its discriminant is $b^2 - 4ac = (4)^2 - 4(-1)(5) = 36$, which is positive.

The equation has two real roots.
The parabola has two x-intercepts.

b. Since the coefficient of x^2 is negative, the parabola opens downward. Its vertex must be above the x-axis (otherwise, the parabola would not intersect the x-axis in two points).

Without drawing the graph of the given equation, determine (a) how many x-intercepts the parabola has and (b) whether its vertex lies above, below, or on the x-axis.

16. $y = x^2 - x - 6$
17. $y = -x^2 + 3x + 4$
18. $y = x^2 + 9 - 6x$
19. $y = 2x^2 + x + 3$
20. $y = 5x - 2 + 3x^2$
21. $y = 4x^2 + 4x + 1$

Mixed Review Exercises

Simplify. Assume no denominator equals zero.

1. $\dfrac{\sqrt{2} - 1}{\sqrt{3}}$
2. $\sqrt{\dfrac{8c^3}{3}} \cdot \sqrt{\dfrac{6c}{25}}$
3. $3\sqrt{27} - 10\sqrt{3} + \sqrt{12}$
4. $\dfrac{2x + 6}{x^3 + 5x^2 + 6x}$
5. $\dfrac{x}{x - 1} + \dfrac{3}{2x + 2}$
6. $(3.1 \cdot 10^3)(4.2 \cdot 10^2)$

Find the vertex and the axis of symmetry of the graph of each equation.

7. $y = 2x^2$
8. $y = x^2 + 6x + 9$
9. $y = 2x^2 + 5$
10. $y = x^2 - 2x$
11. $y = -x^2 + 4x$
12. $y = x^2 + 4x - 5$