Cameron Tinker
Math 122
3/14/19

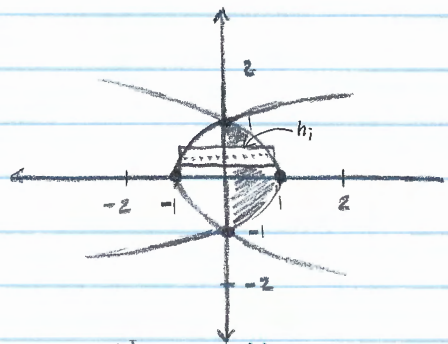
Quiz 12 (pg. 434 #11, 14, 20, 24, 49)

11) Sketch the region enclosed by the given curves and find its area. Decide whether to intergate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

$x = 1 - y^2$, $x = y^2 - 1$

$x = 1$ $\quad x = -1$

$y = \pm 1$ $\quad \Delta y = \frac{2}{n}$



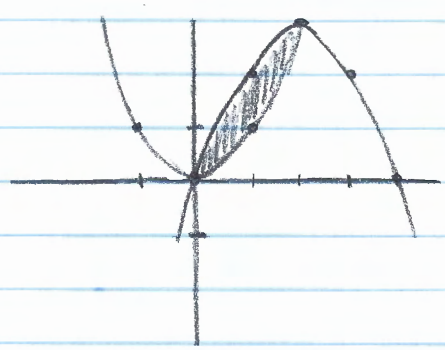
$A = \int_a^b [x_R - x_L]\,dy$

$A = 2\int_0^1 [(1-y^2)-(y^2-1)]\,dy = 2\int_0^1 [2-2y^2]\,dy = 4\int_0^1 [1-y^2]\,dy = 4[y - \frac{1}{3}y^3]_0^1$

$A = 4[1 - \frac{1}{3}] = \boxed{\frac{8}{3}}$

14) Sketch the region enclosed by the given curves and find the area.

$y = x^2$, $y = 4x - x^2$



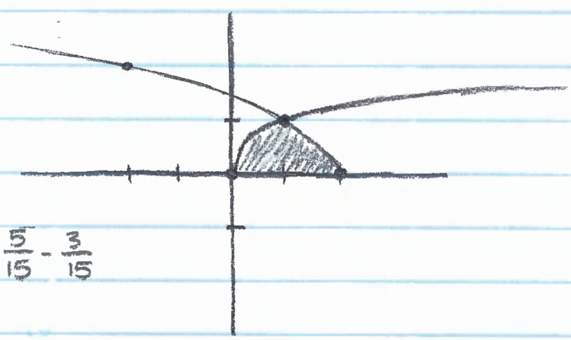
$A = \int_0^2 [y_H - y_L]\,dx = \int_0^2 [(4x - x^2) - x^2]\,dx$

$= \int_0^2 (4x - 2x^2)\,dx = 2\int_0^2 (2x - x^2)\,dx$

$= 2[x^2 - \frac{1}{3}x^3]_0^2 = 2[4 - \frac{8}{3}] = \boxed{\frac{8}{3}}$

20) $x = y^4$, $y = \sqrt{2-x}$, $y = 0$

$x = y^4$, $x = 2 - y^2$, $y = 0$

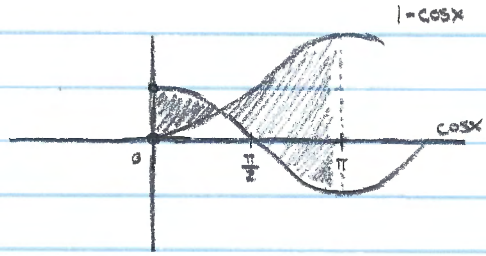


$A = \int_0^1 [x_R - x_L]\,dy = \int_0^1 [(2-y^2) - y^4]\,dy$

$= [2y - \frac{1}{3}y^3 - \frac{1}{5}y^5]_0^1 = 2 - \frac{1}{3} - \frac{1}{5} = \frac{30}{15} - \frac{5}{15} - \frac{3}{15}$

$A = \boxed{\frac{22}{15}}$

24) $y = \cos x$, $y = 1 - \cos x$, $0 \le x \le \pi$



y	cos x	1 - cos x
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$1 - \frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$1 - \frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{1}{2}$

$$A = \int_0^{\frac{\pi}{3}} [\cos x - (1 - \cos x)]\,dx + \int_{\frac{\pi}{3}}^{\pi} [(1 - \cos x) - \cos x]\,dx$$

$$= \int_0^{\frac{\pi}{3}} [2\cos x - 1]\,dx + \int_{\frac{\pi}{3}}^{\pi} [1 - 2\cos x]\,dx = [2\sin x - x]_0^{\frac{\pi}{3}} + [x - 2\sin x]_{\frac{\pi}{3}}^{\pi}$$

$$= \left[2 \cdot \frac{\sqrt{3}}{2} - \frac{\pi}{3}\right] - [\pi - 0] - \left[\frac{\pi}{3} - 2 \cdot \frac{\sqrt{3}}{2}\right] = \sqrt{3} - \frac{\pi}{3} - \pi - \frac{\pi}{3} + \sqrt{3} = \boxed{2\sqrt{3} + \frac{\pi}{3}}$$

49) A cross-section of an airplane wing is shown. Measurements of thickness of the wing, in cm, at 20cm intervals are 5.8, 20.3, 26.7, 29.0, 27.6, 27.3, 23.8, 20.5, 15.1, 8.7, and 2.8. Use the midpoint rule to estimate the area of the wing's cross section.

x	y
0	5.8
20	20.3
40	26.7
60	29.0
80	27.6
100	27.3
120	23.8
140	20.5
160	15.1
180	8.7
200	2.8

pg. 384 Midpoint Rule: $\int_a^b f(x)\,dx \approx \sum f(\bar{x}_i)\Delta x = \Delta x[f(\bar{x}_1) + \dots + f(\bar{x}_n)]$

$\Delta x = \frac{b-a}{n}$, $\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i)$

Let $n = 5$, then $\Delta x = \frac{200}{5} = 40$

$\therefore$ midpoints at 20, 60, 100, 140, 180

$A \approx \Delta x[f(x_{20}) + f(x_{60}) + f(x_{100}) + f(x_{140}) + f(x_{180})]$

$A \approx 40[20.3 + 29.0 + 27.3 + 20.5 + 8.7]$

$A \approx 40[105.8] = \boxed{4232\text{ cm}^2}$