

Memorize the derivative formulas given below in Table 1 and the differentiation rules that follow it.

TABLE 1. **Basic Derivative Formulas**

$f(x)$	$f'(x)$
$c$ ( $c$ , a constant)	0
$x$	1
$x^n$ ( $n$ , any real number)	$nx^{n-1}$
$e^x$	$e^x$
$b^x$	$b^x \ln b$
$\ln  x $	$\frac{1}{x}$
$\log_b  x $	$\frac{1}{x \ln b}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$

Let  $f$  and  $g$  denote functions that are differentiable on an interval  $(a, b)$ . Let  $c$  denote a constant. For  $x \in (a, b)$ , we have the following differentiation rules:

### Constant Multiple Rule

$$\frac{d}{dx}[cf(x)] = cf'(x)$$

In words: *The derivative of a constant times a function is equal to the constant times the derivative of the function.*

### Sum & Difference Rules

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

In words: *The derivative of a sum (difference) is equal to the sum (difference) of the derivatives.*

### Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

In words: *The derivative of a product of two functions is equal to the first function times the derivative of the second function plus the second function times the derivative of the first function.*

### Quotient Rule

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \quad (\text{provided } g(x) \neq 0)$$

Mnemonic: *Low d'high minus high d'low over the square of what's below.*

### Chain Rule

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

In words: *The derivative of a composite function is equal to the derivative of the outer function evaluated at the inner function times the derivative of the inner function.*

### Chain Rule expressed using Leibniz notation

If  $y = f(g(x))$  is rewritten in the form  $y = f(u)$ , where  $u = g(x)$ , then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

### Power Rule combined with the Chain Rule

Let  $n$  be a real number. Let  $u = g(x)$  be a differentiable function.

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} \frac{d}{dx}g(x) \quad \text{or} \quad \frac{d}{dx}u^n = \frac{d}{du}u^n \cdot \frac{du}{dx} = nu^{n-1} \frac{du}{dx}$$

### Differentiation Formulas combined with the Chain Rule

When the argument  $x$  of a function listed in Table 1 is replaced with a differentiable function  $u = g(x)$ , the chain rule must be combined with the formula, such as above, and as follows:

$$\frac{d}{dx}e^u = e^u \frac{du}{dx}, \quad \frac{d}{dx} \ln|u| = \frac{1}{u} \frac{du}{dx}, \quad \frac{d}{dx} \sin u = \cos u \frac{du}{dx}, \quad \frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx},$$

$$\frac{d}{dx} \sec u = \sec u \tan u \frac{du}{dx}, \quad \text{and so on.}$$