Quiz 20

9a p524 Use the Trapezoidal Rule with $n=10$ to approximate the integral $\int_0^2 \frac{e^x}{1+x^2}\,dx$.

Soln. Using the Trapezoidal Rule,

$$T_n(f) = \frac{h}{2}\left[f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)\right]$$

with $f(x) = \frac{e^x}{1+x^2}$, $n=10$, $a=0$, $b=2$, and $h = \frac{b-a}{n} = \frac{2}{10} = 0.2$, we have

$$T_{10}(f) = \frac{0.2}{2}\left[f(0) + 2\sum_{i=1}^{9} f(x_i) + f(2)\right]$$

where $x_i = a + ih = 0.2i$. Thus,

$$T_{10}(f) = 0.1\left[1 + 2\sum_{i=1}^{9} \frac{e^{0.2i}}{1+0.04i^2} + \frac{e^2}{5}\right]$$

$$= 2.660833$$

This is what I want to see on the final exam.

9b p524 Use the Midpoint Rule with $n=10$ to approximate the above integral.

Soln. Using $M_n(f) = \sum_{i=1}^{n} f(m_i)\Delta x$ with $m_i = a + (i-\frac{1}{2})\Delta x = 0.2(i-0.5) = 0.2i - 0.1$

we have

$$M_{10}(f) = \sum_{i=1}^{10} f(m_i)\Delta x = 0.2\sum_{i=1}^{10}\frac{e^{m_i}}{1+m_i^2} = 0.2\sum_{i=1}^{10}\frac{e^{(0.2i-0.1)}}{1+(0.2i-0.1)^2}$$

Therefore,

$$\int_0^2 \frac{e^x}{1+x^2}\,dx \approx 0.2\sum_{i=1}^{10}\frac{e^{(0.2i-0.1)}}{1+(0.2i-0.1)^2} = 2.664377$$

Remark. A CAS or fancy calculator gives 2.663193

This is the answer I want to see on the final.

18a p524 Use the Trapezoidal Rule with $n = 10$ to estimate $\int_0^1 \sqrt{x+x^3}\,dx$.

Soln. Use $T_n(f) = \frac{h}{2}\left[f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)\right]$

with $a=0$, $b=1$, $h = \frac{b-a}{n} = \frac{1}{10} = 0.1$, $f(x) = \sqrt{x+x^3}$, $x_i = a+ih = 0.1i$.

$$T_{10}(f) = \frac{0.1}{2}\left[f(0) + 2\sum_{i=1}^{9} f(x_i) + f(b)\right] = 0.05\left[f(0) + 2\sum_{i=1}^{9}\sqrt{x_i+x_i^3} + f(1)\right]$$

$$= 0.05\left[\sqrt{2} + 2\sum_{i=1}^{9}\sqrt{0.1i + (0.1i)^3}\right]$$

So,

$$\int_0^1 \sqrt{x+x^3}\,dx \approx \boxed{0.5\left[\sqrt{2} + 2\sum_{i=1}^{9}\sqrt{0.1i+(0.1i)^3}\right]} = \underline{0.787092}$$

* This is what I want to see on the final.

18b p524 Use the Midpoint Rule with $n=10$ to approximate the above.

Soln. $m_i = a + (i-0.5)h = 0.1(i-0.5) = 0.1i - 0.05$

$$M_{10}(f) = \sum_{i=1}^{10} f(m_i)\,\Delta x = 0.1\sum_{i=1}^{10}\sqrt{m_i + m_i^3}$$

$$= 0.1\sum_{i=1}^{10}\sqrt{0.1i - 0.05 + (0.1i - 0.05)^3}.$$

Thus,

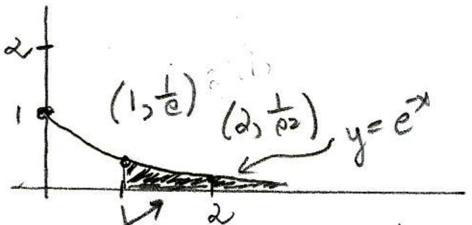
$$\int_0^1 \sqrt{x+x^3}\,dx \approx 0.1\sum_{i=1}^{10}\sqrt{0.1i-0.05+(0.1i-0.05)^3} = \underline{0.793821}$$

Remark. Using a fancy calculator or a CAS like Mathematica or Maple,

$$\int_0^1 \sqrt{x+x^3}\,dx = \underline{0.792486}$$

$\boxed{41\ p535}$ Sketch the region $S = \{(x,y) \mid x \geq 1,\ 0 \leq y \leq e^{-x}\}$ and find its area.

Soln.



$f(x) = e^{-x}$

$f'(x) = -e^{-x} < 0 \Rightarrow$ decreasing

$f''(x) = e^{-x} > 0 \Rightarrow$ concave upward

$f(1) = \frac{1}{e} \approx 0.367$

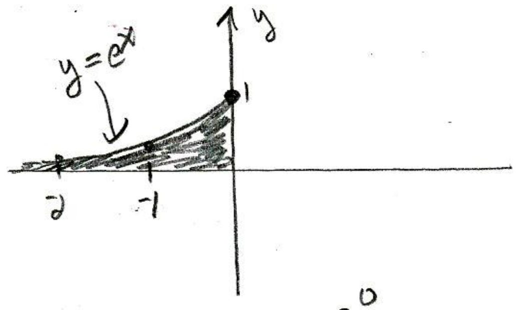
$f(2) = \frac{1}{e^2} \approx 0.135$

$$\text{Area} = \int_1^\infty e^{-x}dx = \lim_{b\to\infty} \int_1^b e^{-x}dx = \lim_{b\to\infty} \left[-e^{-x}\right]_1^b = \lim_{b\to\infty} \left[-\frac{1}{e^b} + \frac{1}{e}\right] = \frac{1}{e}$$

$$\text{Area} = \frac{1}{e}$$

$\boxed{42\ p535}$ Sketch $S = \{(x,y) \mid x \leq 0,\ 0 \leq y \leq e^{x}\}$ and find its area.

Let $f(x) = e^x$. $f'(x) = e^x \Rightarrow f$ is increasing. $f''(x) = e^x > 0 \Rightarrow f$ is concave upward



$$\int_{-\infty}^0 e^x dx = \lim_{a\to-\infty} \int_a^0 e^x dx = \lim_{a\to-\infty} e^x\Big|_a^0 = \lim_{a\to-\infty} \left(e^0 - e^a\right)$$

$$= \lim_{a\to-\infty} \left(1 - e^a\right) = 1.$$

$\therefore$ Area $= 1$