Math 112 - Practice for Test 3
November 21, 2018

Name Answers

Grade: $\frac{\quad}{80} = \_\_\_\_$

No calculators. Circle or box-in answers.

1. Use differentiation rules to find the derivative of each of the following functions. Simplify and label each derivative appropriately. Circle or box-in your final answer.

(a) $f(x) = x^{14} - 5x^3 \sin\left(\frac{x}{2}\right)$ (6 pts.)

$$f'(x) = 14x^3 - \frac{5}{2}x^3 \cos\left(\frac{x}{2}\right) - 15x^2 \sin\left(\frac{x}{2}\right)$$

(b) $y = \dfrac{x^2 - x + 3}{x}$ (6 pts.)

$$y' = 1 - \frac{3}{x^2}$$

(c) $g(t) = \dfrac{\tan \pi t}{1 + t^2}$ (6 pts.)

$$g'(t) = \frac{\pi(1+t^2)\sec^2(\pi t) - 2t\tan(\pi t)}{(1+t^2)^2}$$

(d) $z = 4(\sqrt[3]{x})^2 - \frac{2}{3}e^{-8x^2} + 5\,e^4.$ (6 pts.)

$$z' = \frac{8}{3}x^{-\frac{1}{3}} + \frac{32}{3}xe^{-8x^2} = \frac{8}{3\sqrt[3]{x}} + \frac{32x}{3e^{8x^2}}$$

(e) $y = \sqrt{5 + \ln 4x}$ (6 pts.)

$$y' = \frac{1}{2x\sqrt{5+\ln 4x}}$$

(f) $\phi(t) = \log_2 t - \frac{5}{6}\cos 3t$ (6 pts.)

$$\phi'(t) = \frac{1}{t\ln 2} + \frac{5}{2}\sin 3t$$

(g) $y = 4^x + \ln|\sec x|$ (6 pts.)

$$y' = 4^x \ln 4 + \tan x$$

2. Let $B(t)$ be the number of US \$20 bills in circulation at time t. The following table gives values of this function from 1990 to 2010, as of December 31, in billions.

t	1990	1995	2000	2005	2010
$B(t)$	3.45	4.21	4.93	5.77	6.53

Estimate the value of the derivative $B'(2000)$ and interpret. (8 pts.)

$B'(t)$ is the rate at which the number of \$20 bills is changing wrt time in years.

$$B'(2000) \approx \frac{B(2005) - B(1995)}{2005 - 1995} = \frac{5.77 - 4.21}{10} = \frac{1.56 \leftarrow \text{billion \$20 bills}}{10 \leftarrow \text{years}}$$

So, $B'(2000) \approx 0.156$ billion bills per year

Or, $B'(2000) \approx 156$ million twenty-dollar bills per year.

3. Find the equation of the tangent line to the curve $y = 1 - \cos(x/2)$ when $x = \pi$. (6 pts.)

$$y-1 = \frac{1}{2}(x-\pi)$$

4. Show how to find $\lim_{x \to 0} \dfrac{\sin 3x \sin 4x}{x^2}$ using a well-know limit that was derived in class. (6 pts.)

$$\lim_{x \to 0} \frac{\sin 3x \sin 4x}{x^2} = 12$$

5. A table of values for f, g, f', and g' is given in the following table. (8 pts.)

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

(a) If $h(x) = f(g(x))$, find $h'(1)$.

$$h'(1) = 30$$

(b) If $H(x) = g(f(x))$, find $H'(1)$.

$$H'(1) = 36$$

6. A ladder 10 feet long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall and let x be the the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when $\theta = \pi/3$? (5pts.)

$$\left.\frac{dx}{d\theta}\right|_{x=\frac{\pi}{3}} = 10\cos\frac{\pi}{3} = 5\,\frac{ft}{rad}$$

7. Use logarithmic differentiation to find the derivative of $y = (\tan x)^{1/x}$. (5pts.)

$$y' = (\tan x)^{\frac{1}{x}} \cdot \frac{1}{x}\left(\sec x \csc x - \frac{\ln \tan x}{x}\right)$$

Extra Credit. (4 pts.) Use the definition of the derivative to determine the value of $\lim_{x\to 0} \dfrac{\ln(1+x)}{x}$.

Since $\ln 1 = 0$,

$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = \lim_{x\to 0} \frac{\ln(1+x) - \ln 1}{x} = \lim_{h\to 0} \frac{\ln(1+h) - \ln 1}{h}$$

$$= \left.\frac{d}{dx}\ln x\right|_{x=1} = \left.\frac{1}{x}\right|_{x=1} = \boxed{1}$$