

9-7 Puzzle Problems

Objective: To use systems of equations to solve digit, age, and fraction problems.

Example 1 (Digit problem)

The sum of the digits in a two-digit number is 10. The new number obtained when the digits are reversed is 18 more than the original number. Find the original number.

Solution

Step 1 The problem asks for the original number.

Step 2 Let t = the tens digit and u = the units digit of the original number.

	Tens	Units	Value
Original number	t	u	$10t + u$
Number with digits reversed	u	t	$10u + t$

Step 3 Use the facts of the problem to write two equations.

$$t + u = 10 \quad \text{Sum of the digits of the original number is 10.}$$

$$\begin{aligned} (10u + t) - (10t + u) &= 18 & \left\{ \begin{array}{l} \text{Difference between new number} \\ \text{and original number is 18.} \end{array} \right. \\ 10u + t - 10t - u &= 18 \\ 9u - 9t &= 18 \\ 9(u - t) &= 18 \\ u - t &= 2 \end{aligned}$$

Step 4 $u + t = 10$ $\left\{ \begin{array}{l} \text{Write the two equations as a system and} \\ \text{solve for one variable.} \end{array} \right.$

$$\begin{array}{r} u + t = 10 \\ u - t = 2 \\ \hline 2u = 12 \\ u = 6 \end{array}$$

$$\begin{array}{r} u - t = 2 \\ 6 - t = 2 \\ t = 4 \end{array} \quad \text{Substitute 6 for } u \text{ in the second equation.}$$

Step 5 The check is left for you.

The original number is 46.

Solve by using a system of two equations in two variables.

- The sum of the digits in a two-digit number is 7. The new number obtained when the digits are reversed is 27 less than the original number. Find the original number.
- A two-digit number is seven times the sum of its digits. The tens digit is 3 more than the units digit. What is the number?

9-7 Puzzle Problems (continued)

Example 2 (Age problem) Chan is three years older than Myra. Six years ago Chan was twice as old as Myra was. Find their ages now.

Solution

Steps 1, 2 Let c = Chan's age now and let m = Myra's age now.

Step 3 Use the facts of the problem to write two equations:

$$\begin{aligned} c &= m + 3 && \{\text{now} \\ c - 6 &= 2(m - 6) && \{\text{six years ago} \end{aligned}$$

Age	Now	6 years ago
Chan	c	$c - 6$
Myra	m	$m - 6$

Step 4 Simplify the equations and solve the system: $m = 9, c = 12$

Step 5 The check is left for you. Chan is 12 years old now and Myra is 9.

Example 3 (Fraction problem) The denominator of a fraction is 4 more than the numerator. If 2 is subtracted from each, the value of the new fraction is $\frac{1}{5}$. Find the original fraction.

Solution

Steps 1, 2 Let n = the numerator and d = the denominator of the original fraction.

Step 3 Use the facts of the problem to write two equations.

$$\begin{aligned} d &= n + 4 \\ \frac{n - 2}{d - 2} &= \frac{1}{5}, \text{ or } 5(n - 2) = d - 2 \end{aligned}$$

Step 4 Simplify the equations and solve the system: $n = 3, d = 7$

Step 5 The check is left to you. The original fraction $\frac{n}{d}$ is $\frac{3}{7}$.

Solve by using a system of two equations in two variables.

- Max is 5 years older than Paulette. Next year he will be twice as old as she will be. How old is each now?
- Gloria is 20 years older than Reggie. Five years ago she was five times as old as he was. How old is each now?
- The denominator of a fraction is 7 more than the numerator. If 5 is added to both the numerator and denominator, the value of the resulting fraction is $\frac{1}{2}$. What is the original fraction?
- The denominator of a fraction is 1 more than the numerator. If the numerator is decreased by 1, the value of the resulting fraction is $\frac{3}{4}$. What is the original fraction?

Mixed Review Exercises

Solve.

$$1. \frac{x + 3}{2} - \frac{x}{3} = \frac{5}{6} \quad 2. \frac{3n + 2}{5} = \frac{n - 2}{3} \quad 3. \frac{3 + c}{2 + c} = \frac{3}{4} \quad 4. -\frac{1}{4}x = 12$$