Quiz 18

1a Write out the form of the partial fraction decomposition of

$$\frac{4+x}{(1+2x)(3-x)}.$$

Soln.

$$\boxed{\frac{4+x}{(1+2x)(3-x)} = \frac{A}{1+2x} + \frac{B}{3-x}}$$

1b $\dfrac{1-x}{x^3+x^4}$

$$\boxed{\frac{1-x}{x^3(1+x)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{1+x}}$$

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4 Write out the form of the p.f.d.

(a) $\dfrac{x^4-2x^3+x^2+2x-1}{x^2-2x+1}$

$$\frac{x^4-2x^3+x^2+2x-1}{x^2-2x+1} = \frac{x^2(x^2-2x+1)-x^2+x^2+2x-1}{x^2-2x+1}$$

$$= \frac{x^2(x^2-2x+1)+2x-1}{x^2-2x+1} = x^2 + \frac{2x-1}{(x-1)^2}.$$

Or use long division

$$\begin{array}{r|l} & x^2 \\ \hline x^2-2x+1 & x^4-2x^3+x^2+2x-1 \\ & x^4-2x^3+x^2 \\ \hline & 2x-1 \end{array}$$

$$\Rightarrow \frac{x^4-2x^3+x^2+2x-1}{x^2-2x+1} = x^2 + \frac{2x-1}{x^2-2x+1}$$

$$\therefore \boxed{\frac{x^4-2x^3+x^2+2x+1}{x^2-2x+1} = x^2 + \frac{A}{x-1} + \frac{B}{(x-1)^2}}$$

4b $\dfrac{x^2-1}{x^3+x^2+x}$

Soln. $\dfrac{x^2-1}{x^3+x^2+x} = \dfrac{x^2-1}{x(x^2+x+1)}$

Since x^2+x+1 is irreducible ($b^2-4ac = 1^2-4(1)(1) < 0$), the pfd is

$$\frac{x^2-1}{x^3+x^2+x} = \frac{A}{x} + \frac{Bx+C}{x^2+x+1}$$

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9 p501 Evaluate $\displaystyle\int \frac{5x+1}{(2x+1)(x-1)}\,dx$.

Soln. $\dfrac{5x+1}{(2x+1)(x-1)} = \dfrac{A}{2x+1} + \dfrac{B}{x-1}$

$\Rightarrow A(x-1) + B(2x+1) = 5x+1$

Let $x=1$: $3B = 6 \Rightarrow B = 2$

Let $x=-\frac{1}{2}$: $-\frac{3}{2}A = -\frac{5}{2}+1 \Rightarrow -\frac{3}{2}A = -\frac{3}{2} \Rightarrow A = 1$

$$\therefore \int \frac{x^2-1}{x^3+x^2+x}\,dx = \int \frac{dx}{2x+1} + 2\int \frac{dx}{x-1}$$

$$= \frac{1}{2}\ln|2x+1| + 2\ln|x-1| + C$$

10 p 501 Evaluate $\int \frac{y}{(y+4)(2y-1)}\,dy$.

Soln. $\frac{y}{(y+4)(2y-1)} = \frac{A}{y+4} + \frac{B}{2y-1}$

$A(2y-1) + B(y+4) = y$

Let $y=-4$. Then $-9A = -4$. $A = 4/9$

Let $y = \frac{1}{2}$. Then $\frac{9}{2}B = \frac{1}{2} \Rightarrow B = \frac{1}{9}$

So, $\int \frac{y\,dy}{(y+4)(2y-1)} = \frac{4}{9}\int \frac{dy}{y+4} + \frac{1}{9}\int \frac{dy}{2y-1}$

$= \frac{4}{9}\ln|y+4| + \frac{1}{18}\ln|2y-1| + C$