Example of Simpson's Rule

>

> *restart*

> $with(Student[Calculus1]):$

> $Digits := 9$

$$Digits := 9 \quad (1)$$

Example 2. Use **Simpson's Rule** to approximate $\int_0^3 e^{-x^2}\,dx$ with $N = 6$ and 50 subintervals.

> $f := x \to e^{-x^2}$

$$f := x \to e^{-x^2} \quad (2)$$

> $a := 0; b := 3$

$$a := 0$$
$$b := 3 \quad (3)$$

>

Approximate the integral by partitioning $[0, 3]$ into <u>6 subintervals</u> of equal width:

> $N := 6$

$$N := 6 \quad (4)$$

> $h := \frac{b-a}{N}$

$$h := \frac{1}{2} \quad (5)$$

> $\frac{h}{3}\left(f(a) + 4\cdot Sum\left(f(a + (2\cdot i - 1)\cdot h), i = 1 .. \frac{N}{2}\right) + 2\cdot Sum\left(f(a + 2\cdot i\cdot h), i = 1 .. \frac{N}{2} - 1\right) + f(b)\right)$

$$\frac{1}{6} + \frac{2}{3}\sum_{i=1}^{3} e^{-\left(i-\frac{1}{2}\right)^2} + \frac{1}{3}\sum_{i=1}^{2} e^{-i^2} + \frac{1}{6}e^{-9} \quad (6)$$

>

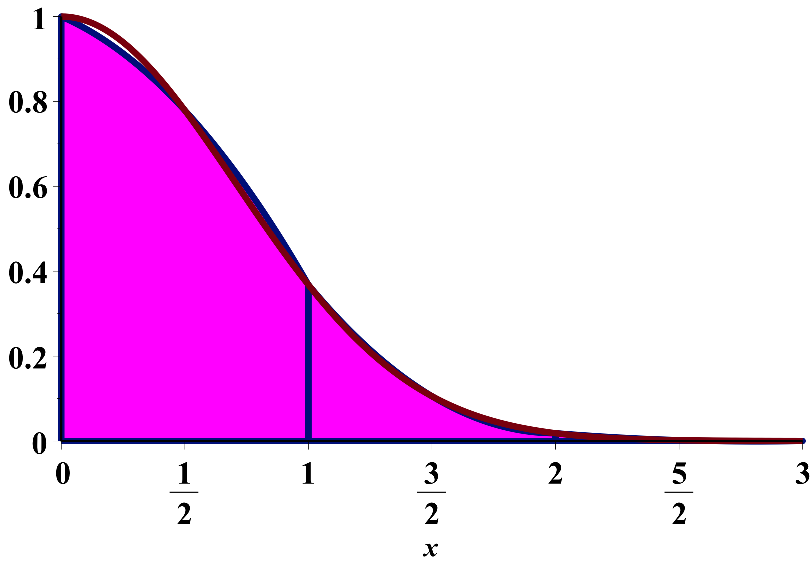
> $evalf(\%)$

$$0.886172570 \quad (7)$$

>

> $ApproximateInt\Big(f(x), x = a..b, output = plot, partition = N, partitiontype = normal,$
$method = simpson, boxoptions = [filled = [color = \text{"Magenta"}, transparency = .9]],$

$tickmarks = \left(\left[spacing\left(\frac{b-a}{N}\right), default\right]\right), thickness = 3, font = [Roman, bold, 12],$
$labelfont = [Roman, 10]\Big)$



An approximation of $\int_0^3 f(x)\,dx$ using Simpson's rule, where $f(x) = e^{-x^2}$ and the partition is uniform. The approximate value of the integral is 0.886172570. Number of subintervals used: 3.

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Approximate the integral by partitioning $[0, 3]$ into <u>50 subintervals</u> of equal width:

> $N := 50$

$$N := 50 \tag{8}$$

> $h := \frac{b-a}{N}$

$$h := \frac{3}{50} \tag{9}$$

> $\frac{h}{3}\left(f(a)+4\cdot Sum\left(f(a+(2\cdot i-1)\cdot h), i=1..\frac{N}{2}\right)+2\cdot Sum\left(f(a+2\cdot i\cdot h), i=1..\frac{N}{2}-1\right)+f(b)\right)$

$$\frac{1}{50}+\frac{2}{25}\sum_{i=1}^{25}\mathrm{e}^{-\left(\frac{3}{25}i-\frac{3}{50}\right)^2}+\frac{1}{25}\sum_{i=1}^{24}\mathrm{e}^{-\frac{9}{625}i^2}+\frac{1}{50}\mathrm{e}^{-9} \qquad \textbf{(10)}$$

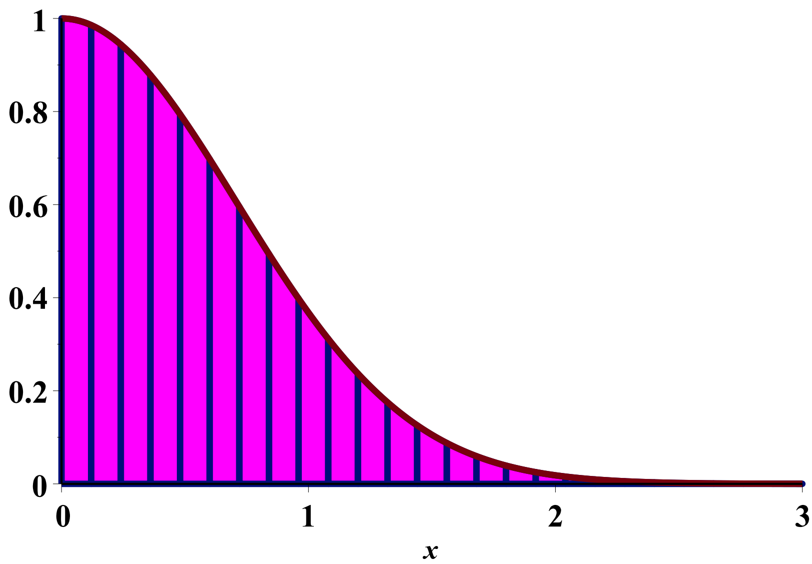
>

> *evalf*(%)

$$0.886207346 \qquad \textbf{(11)}$$

>

> *ApproximateInt*(*f*(*x*), *x* = *a* ..*b*, *output* = *plot*, *partition* = *N*, *partitiontype* = *normal*, *method* = *simpson*, *boxoptions* = [*filled* = [*color* = "Magenta", *transparency* = .9]], *tickmarks* = ([*spacing*(1), *default*]), *thickness* = 3, *font* = [*Roman*, *bold*, 12], *labelfont* = [*Roman*, 10])



An approximation of $\int_0^3 f(x)\,dx$ using Simpson's rule, where $f(x) = e^{-x^2}$ and the partition is uniform. The approximate value of the integral is 0.886207347. Number of subintervals used: 25.

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Finally, let us compare the above approximations with the value obtained using the *error function*, which is defined by

$$\mathrm{erf}(x) := \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt.$$

>

Consequently, $\int_0^x e^{-t^2}\,dt = \frac{\sqrt{\pi}}{2}\mathrm{erf}(3)$. Since the error function is built into *Maple*, we have

> $\frac{\sqrt{\pi}}{2}\mathrm{erf}(3)$

$$\frac{1}{2}\sqrt{\pi}\,\mathrm{erf}(3) \quad \textbf{(12)}$$

> *evalf*(%, 12)

$$0.886207348260 \quad \textbf{(13)}$$

**** End ****

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