Quiz 13

p 421. Are the following statements true or false?

[2] If f and g are continuous on $[a,b]$, then

$$\int_a^b [f(x)\, g(x)]\, dx = \int_a^b f(x)dx \cdot \int_a^b g(x)\,dx.$$

This is $\boxed{\text{false.}}$ Here is a counterexample.

Let $a=0$, $b=2$, $f(x)=1$, $g(x)=2x$. Then

$$\int_0^2 [f(x)\, g(x)]\, dx = \int_0^2 2x\, dx = x^2\Big|_0^2 = 4$$

whereas

$$\int_0^2 f(x)\,dx \cdot \int_0^2 g(x)\,dx = \int_0^2 dx \cdot \int_0^2 2x\,dx = x\Big|_0^2 \cdot x^2\Big|_0^2 = 2(4) = 8.$$

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[6] If f' is continuous on $[1,3]$, then

$$\int_1^3 f'(v)\,dv = f(3) - f(1).$$

Soln. An antiderivative of f' is f. So it follows from the FTOC-2 (see p. 396) that

$$\int_1^3 f'(v)\,dv = f(v)\Big|_1^3 = f(3) - f(1).$$

Could also say it is true by the Net Change Theorem.

So, the statement is $\boxed{\text{true}}$.

$\boxed{8\,p421}$ If f and g are differentiable and $f(x) \geq g(x)$ for $a < x < b$, then $f'(x) \geq g'(x)$ for $a < x < b$.

$\boxed{\text{False}}$ A counterexample is $f(x) = 1$ and $g(x) = x$ on $(0,1)$. Although $f(x) \geq g(x)$ on $(0,1)$, $f'(x) < g'(x)$ on $(0,1)$ since $f'(x) = 0$ and $g'(x) = 1$.

$\boxed{14\,p\,421}$ If $\int_0^1 f(x)\,dx = 0$, then $f(x) = 0$ for $0 \leq x \leq 1$.

$\boxed{\text{False}}$ Counterexample: Let $f(x) = 2x-1$. Then

$$\int_0^1 f(x)\,dx = \int_0^1 (2x-1)\,dx = \left[x^2 - x\right]_0^1 = 0.$$

$\boxed{\#2\,p476}$ Evaluate $\int \sqrt{x}\ln x\,dx$.

Soln. LIATE

$u = \ln x$ $\quad dv = \sqrt{x}\,dx$

$$u = \ln x \quad (+1) \quad dv = x^{1/2}\,dx$$

$$du = \frac{dx}{x} \quad (-1) \quad v = \frac{2}{3}x^{3/2}$$

$$\int \sqrt{x}\ln x\,dx = \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int \frac{x^{3/2}}{x}\,dx$$

$$= \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int x^{\frac{1}{2}}\,dx$$

$$= \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\cdot\frac{2}{3}x^{3/2} + C.$$

Thus,

$$\int \sqrt{x}\ln x\,dx = \frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C$$

#3 p 476 Evaluate $\int x\cos(5x)\,dx$.

Soln. LIATE

$u = x \quad dv = \cos(5x)\,dx$

$u = x \qquad (+1) \qquad dv = \cos(5x)\,dx$

$du = dx \qquad (-1) \qquad v = \frac{1}{5}\sin(5x)$

$$\int x\cos(5x)\,dx = \frac{1}{5}x\sin(5x) - \frac{1}{5}\int \sin(5x)\,dx$$

$$= \frac{x}{5}\sin(5x) + \frac{1}{25}\cos(5x) + C$$

#4 p 476 Evaluate $\int y e^{0.2y}\,dy$.

Soln. LIATE

$u = y \quad dv = e^{0.2y}\,dy$

$u = y \qquad (+1) \qquad dv = e^{0.2y}\,dy$

$du = dy \qquad (-1) \qquad v = 5e^{0.2y}$

$$\int y e^{0.2y}\,dy = 5ye^{0.2y} - 5\int e^{0.2y}\,dy = 5ye^{0.2y} - 25e^{0.2y} + C$$

Extra Credit (#6 p 476)

Evaluate $\int (x-1)\sin(\pi x)\,dx$.

LIATE

$u = x-1 \quad \sin(\pi x)\,dx$

$u = x-1 \qquad (+1) \qquad dv = \sin(\pi x)\,dx$

$du = dx \qquad (-1) \qquad v = -\frac{1}{\pi}\cos(\pi x)$

$$\int (x-1)\sin(\pi x)\,dx = -\frac{1}{\pi}(x-1)\cos(\pi x) + \frac{1}{\pi}\int \cos(\pi x)\,dx$$

$$= \frac{1-x}{\pi}\cos(\pi x) + \frac{1}{\pi^2}\sin(\pi x) + C.$$