

1-7 Solving Equations in One Variable

Objective: To solve certain equations in one variable.

Vocabulary

Open sentence An equation or inequality that contains one or more variables.

Solution set The set of all values of a variable that make an open sentence true.

A solution is also called a *root*. A solution is said to *satisfy* an equation.

Transformations Changes that produce equivalent equations (ones with the same solution set). They include:

1. Simplifying either side of an equation.
2. Adding the same number to each side of an equation, or subtracting the same number from each side of an equation.
3. Multiplying (or dividing) each side of an equation by the same *nonzero* number.

Solve an equation Transform the equation into a simpler equivalent one whose solution set is easily seen.

Identity An equation that is satisfied by all values of the variable. The solution set of an identity is the set of all real numbers.

Symbols $\stackrel{?}{=}$ (Are they equal?) \therefore (therefore)
 \neq (is not equal to) \emptyset (empty or null set, the set with no members)

Example 1 Solve $3(2x - 3) = 4x + 7$. (The goal is to get x alone on one side.)

Solution

$$\begin{aligned} 6x - 9 &= 4x + 7 && \text{Simplify the left side.} \\ 6x - 9 + 9 &= 4x + 7 + 9 && \text{Add 9 to each side.} \\ 6x &= 4x + 16 \\ 6x - 4x &= 4x + 16 - 4x && \text{Subtract 4x from each side.} \\ 2x &= 16 \\ \frac{2x}{2} &= \frac{16}{2} && \text{Divide each side by 2.} \\ x &= 8 \end{aligned}$$

Check: $3[2(8) - 3] \stackrel{?}{=} 4(8) + 7$ Substitute 8 for x in the *given* equation.

$$3(16 - 3) \stackrel{?}{=} 32 + 7$$

$$3(13) \stackrel{?}{=} 39$$

$$39 = 39 \quad \checkmark$$

\therefore the solution set is $\{8\}$.

Solve. Check your work.

1. $4x - 6 = 2$

2. $6 = 3x + 3$

3. $\frac{1}{2}x - 4 = -2$

4. $7 - \frac{1}{5}y = -2$

5. $48 - 6x = 2x$

6. $x + 2 = 3x - 6$

7. $4(x - 3) = 2x - 6$

8. $3(1 - y) = 3y$

1-7 Solving Equations in One Variable (continued)**Example 2** Solve.

a. $3x - (x - 5) = 2(x + 4)$

b. $3x - (x - 8) = 2(x + 4)$

Solution

a. $3x - x + 5 = 2x + 8$

$2x + 5 = 2x + 8$

$5 = 8$ False!

No value of x will make this statement true. \therefore the solution set is \emptyset .

b. $3x - x + 8 = 2x + 8$

$2x + 8 = 2x + 8$

This is an identity. It is true for all values of x . \therefore the solution set is the set of all real numbers.**Solve. Check your work when there is a single solution.**

9. $5(x - 2) - 3 = -(x + 1)$

10. $2x - 1 = 2(x + 4)$

11. $2(x + 2) = 2x + 4$

12. $4a + 1 = a - 5 - 3a$

13. $1.2(u - 2) = 4.8$

14. $0.4(2r + 3) = 0.6r + 3.6$

15. $3(x - 2) = 5(x - 2)$

16. $\frac{1}{5}(x + 3) = x - 5$

17. $5z - (6 - z) = 4(1 - z)$

18. $11x - 3(4x - 2) = 2(8 + 2x)$

19. $\frac{5k - 3(k - 2)}{4} = -6$

Example 3 Solve the equation $T = c + ct$ for the variable t .**Solution**

$T = c + ct$

$T - c = c + ct - c$

$T - c = ct$

$\frac{T - c}{c} = \frac{ct}{c}$

$t = \frac{T - c}{c}$

Work toward getting t alone on one side.Subtract c from each side.Divide each side by c . (Assume $c \neq 0$.)**Solve each equation for the given variable.**

20. $C = \pi d$ for d

21. $I = prt$ for r

22. $3x - 4y = 8$ for y

23. $P = 2l + 2w$ for l

24. $ax + by + c = 0$ for x

25. $A = 0.5h(a + b)$ for b

Mixed Review Exercises**Simplify.**

1. $5(8 - 6 + 1)$

2. $(-4)^2 - 3^2$

3. $|5 + (-7)|$

4. $6\left(-\frac{1}{2}\right)(-4)\left(-\frac{1}{3}\right)$

5. $\frac{-12 \div 3}{-1 - 1}$

6. $9y - 2(3y - 8)$

7. $3(4ab) - (2a)(-7b)$

8. $(-c)^3(-d)^5$

9. $3(x + 2y) + 4(-2x - y)$

10. $-5p + \frac{2}{3}(9 - 3p) - 2$

11. $\frac{2m - 4}{-2}$