NAME ______________________________ DATE ______________

12–7 Direct and Inverse Variations Involving Squares

Objective: To use quadratic direct variation and inverse variation as a square in problem solving.

Vocabulary

Quadratic direct variation A function of the form

$y = kx^2$, where k is a nonzero constant.

You say that *y varies directly as* x^2 or that *y is directly proportional to* x^2.
If (x_1, y_1) and (x_2, y_2) are ordered pairs of the same quadratic variation, and neither x_1 nor x_2 is zero, then

$$\frac{y_1}{x_1^2} = \frac{y_2}{x_2^2}.$$

Inverse variation as the square A function of the form

$x^2y = k$, where k is a nonzero constant,

or $y = \frac{k}{x^2}$, where $x \neq 0$.

You say that *y varies inversely as* x^2 or *y is inversely proportional to* x^2.

If (x_1, y_1) and (x_2, y_2) are ordered pairs of the function defined by $x^2y = k$, then

$$x_1^2y_1 = x_2^2y_2.$$

Example 1 Given that a varies directly as the square of d, and $a = 18$ when $d = 3$, find the value of d when $a = 8$.

Solution 1 Use $a = kd^2$:

$$18 = 9k$$
$$2 = k$$
$$a = 2d^2$$

For $a = 8$:

$$8 = 2d^2$$
$$4 = d^2$$
$$\pm 2 = d$$

Solution 2

$$\frac{a_1}{d_1^2} = \frac{a_2}{d_2^2}$$
$$\frac{18}{3^2} = \frac{8}{d_2^2}$$
$$18d_2^2 = 72$$
$$d_2^2 = 4$$
$$d_2 = \pm 2$$

CAUTION Remember that in a particular situation you must check each root to see if it makes sense.

Solve. Give roots to the nearest tenth. You may wish to use a calculator.

1. Given that d varies directly as the square of t and $d = 16$ when $t = 2$, find the value of d when $t = 3$.

2. Given that A varies directly as the square of e and $A = 32$ when $e = 2$, find the value of e when $A = 8$.

3. The price of a diamond varies directly as the square of its mass in carats. If a 1.5 carat diamond costs $2700, find the cost of a 3 carat diamond.

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Example 2 Given that h varies inversely as the square of r, and that $r = 3$ when $h = 18$, find the value of r when $h = \frac{3}{10}$.

Solution 1 Use $h = \frac{k}{r^2}$: $18 = \frac{k}{9}$

$162 = k$

$h = \frac{162}{r^2}$

For $h = \frac{3}{10}$: $\frac{3}{10} = \frac{162}{r^2}$

$3r^2 = 1620$

$r^2 = 540$

$r = \pm\sqrt{540}$

$= \pm 6\sqrt{15}$

Solution 2 Use $r_1^2 h_1 = r_2^2 h_2$

$(3)^2 18 = r_2^2\left(\frac{3}{10}\right)$

$162 = \frac{3r_2^2}{10}$

$3r_2^2 = 1620$

$r_2^2 = 540$

$r_2 = \pm\sqrt{540}$

$r_2 = \pm 6\sqrt{15}$

Solve. Give roots to the nearest tenth. You may wish to use a calculator.

4. Given that h varies inversely as the square of r, and that $r = 4$ when $h = 25$, find the value of r when $h = 4$.
5. Given that h varies inversely as the square of r, and that $r = 5$ when $h = 36$, find the value of h when $r = 15$.
6. Light intensity varies inversely as the square of the distance from the source. At 2 m, an intensity scale has a reading of 6. What will be the reading at 6 m?
7. The distance it takes an automobile to stop varies directly as the square of its speed. If the stopping distance for a car traveling at 80 km/h is 175 m, what is the stopping distance for a car traveling at 48 km/h?
8. The height of a cylinder of a given volume is inversely proportional to the square of the radius. A cylinder of radius 4 cm has a height of 12 cm. What is the radius of a cylinder of equal volume whose height is 48 cm?
9. The length of a pendulum varies directly as the square of the time in seconds it takes to swing from one side to the other. If it takes a 100 cm pendulum 1 s to swing from one side to the other, how many seconds does it take a 50 cm pendulum to swing?

Mixed Review Exercises

Solve for the indicated variable. State any restrictions.

1. $S = C(1 - r); r$
2. $d = \frac{a(2t - v)}{2}; v$
3. $A = \frac{h(b + c)}{2}; c$

Write an equation, in standard form, of the line passing through the given points.

4. (2, 5), (4, 6)
5. (0, 2), (−4, 1)
6. (2, 3), (−1, 0)