

6-4 Least Common Denominators

Objective: To express two or more fractions with their least common denominator.

Example 1 Complete: a. $\frac{2}{3} = \frac{?}{15}$ b. $\frac{5}{2a} = \frac{?}{18a^2}$

Solution To write a fraction in a different form, you can multiply the numerator and denominator by the same nonzero number.

a. $\frac{2}{3} = \frac{?}{15}$ ← 3 is multiplied by 5 to get 15.

$\frac{2}{3} = \frac{2 \cdot 5}{3 \cdot 5} = \frac{10}{15}$ ← Therefore, multiply 2 by 5 to get 10.

b. $\frac{5}{2a} = \frac{?}{18a^2}$ ← $2a$ is multiplied by $9a$ to get $18a^2$.

$\frac{5}{2a} = \frac{5 \cdot 9a}{2a \cdot 9a} = \frac{45a}{18a^2}$ ← Therefore, multiply 5 by $9a$ to get $45a$.

Complete.

1. $\frac{2}{3} = \frac{?}{18}$

2. $\frac{3}{5} = \frac{?}{20}$

3. $\frac{5}{8} = \frac{?}{56}$

4. $\frac{2a}{15} = \frac{?}{45}$

5. $\frac{x-2}{3} = \frac{?}{12}$

6. $\frac{2n-3}{5} = \frac{?}{15}$

7. $\frac{8}{15x} = \frac{?}{90x^2}$

8. $\frac{5}{3a} = \frac{?}{9a^3}$

9. $\frac{x}{3y} = \frac{?}{18xy}$

10. $\frac{3n}{4m} = \frac{?}{12m^2n}$

Example 2 Complete: $\frac{2}{x-3} = \frac{?}{(x-3)(x+4)}$

Solution $\frac{2}{x-3} = \frac{?}{(x-3)(x+4)}$ ← $(x-3)$ is multiplied by $(x+4)$.

$\frac{2}{x-3} = \frac{2(x+4)}{(x-3)(x+4)}$ ← Therefore, multiply 2 by $(x+4)$.

Complete.

11. $\frac{6}{n-1} = \frac{?}{(n-1)(n+4)}$

12. $\frac{4}{x+2} = \frac{?}{(x+2)(x-2)}$

13. $\frac{3}{2x-1} = \frac{?}{(2x-1)^2}$

14. $\frac{5y}{x-7} = \frac{?}{(x-7)^2}$

15. $\frac{7}{x-3} = \frac{?}{4x-12}$

16. $\frac{3}{2x+5} = \frac{?}{6x+15}$

17. $\frac{3}{x+2} = \frac{?}{x^2-4}$

18. $\frac{4}{x-1} = \frac{?}{x^2-1}$

19. $\frac{5}{3-y} = \frac{?}{3y-y^2}$

20. $\frac{3x}{2+x} = \frac{?}{2x+x^2}$

6-4 Least Common Denominator (continued)

Example 3 Find the LCD of $\frac{5}{6}$, $\frac{7}{20}$, and $\frac{8}{42}$.

Solution

1. Factor each denominator into prime numbers.

$$6 = 2 \cdot 3 \quad 20 = 2^2 \cdot 5 \quad 42 = 2 \cdot 3 \cdot 7$$

2. Greatest power of 2: 2^2

$$\text{Greatest power of 3: } 3 \quad 2^2 \cdot 3 \cdot 5 \cdot 7 = 420$$

Greatest power of 5: 5

Greatest power of 7: 7 The LCD is 420.

Find the LCD of each group of fractions.

21. $\frac{1}{4}, \frac{5}{6}$

22. $\frac{1}{2}, \frac{3}{8}$

23. $\frac{3}{2}, \frac{2}{5}, \frac{1}{4}$

24. $\frac{2}{3}, \frac{5}{9}, \frac{1}{6}$

25. $\frac{5}{8}, \frac{2}{5}, \frac{4}{3}$

26. $\frac{2}{3}, \frac{3}{4}, \frac{5}{9}$

Example 4 Find the LCD of $\frac{5}{9x - 36}$ and $\frac{4}{5x - 20}$.

Solution

1. Factor each denominator completely. Factor integers into primes.

$$9x - 36 = 9(x - 4) = 3^2(x - 4) \quad 5x - 20 = 5(x - 4)$$

2. Form the product of the greatest power of each factor.

$$3^2 \cdot 5(x - 4) = 45(x - 4)$$

The LCD is $45(x - 4)$.

Find the LCD of each group of fractions.

27. $\frac{a + 2b}{4}, \frac{2b - a}{6}$

28. $\frac{n - 2}{12}, \frac{n + 3}{15}$

29. $\frac{n - 1}{15}, \frac{n + 3}{20}$

30. $\frac{2x + 3}{12}, \frac{x - 4}{8}$

31. $\frac{x + 2y}{25}, \frac{2x + y}{20}$

32. $\frac{x^2 - x - 6}{21}, \frac{x^2 - 9}{35}$

33. $\frac{2}{3r}, \frac{5}{9r^2}$

34. $\frac{5}{xy}, \frac{6}{y^2}$

35. $\frac{11}{m^2n}, \frac{17}{mn^2}$

36. $\frac{3}{2x + 10}, \frac{x}{5x + 25}$

37. $\frac{3a}{a + 1}, \frac{2}{a - 1}$

38. $\frac{3}{a^2 - 4}, \frac{5}{a + 2}$

39. $\frac{x}{x^2 + 3x}, \frac{2x}{x^2 - 3x}$

40. $\frac{7}{n + 3}, \frac{n - 1}{n^2 + n - 6}$

41. $\frac{a + 1}{a - 2}, \frac{a - 5}{a^2 - 5a + 6}$

Mixed Review Exercises

Factor completely.

1. $3n - 9q + 15$

2. $2x^2 - 8$

3. $x^2 - 10x + 16$

4. $x^2 - 5x - 36$

5. $2x^2 - 5x - 3$

6. $x^2 + 14x + 24$

7. $x^2 - 4x - 32$

8. $x^2 + 24x + 144$

9. $n^2 - 6n$