204 #

Mackenzie Sawyer

differentiate

8 $F(x) = (1+x+x^2)^{99}$ Let $u = 1+x+x^2$.

By the Chain Rule,

$$F'(x) = \frac{d}{du}u^{99} \cdot \frac{d}{dx}u$$

$$= 99u^{98} \cdot \frac{d}{dx}u$$

$$= 99(1+x+x^2)^{98} \cdot \frac{d}{dx}(1+x+x^2)$$

$$= 99(1+x+x^2)^{98} \cdot (1+2x)$$

$$= 99(1+x+x^2)^{98}(1+2x)$$

Or, simply remember $\boxed{4}$ on p 200:

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}g'(x).$$

Applying, we have

$$F'(x) = \frac{d}{dx}(1+x+x^2)^{99} = 99(1+x+x^2)^{98}\frac{d}{dx}(1+x+x^2)$$

$$= 99(1+x+x^2)^{98}(1+2x)$$

Differentiate

14 $f(t) = t\sin \pi t$.

Using the Product Rule

$$f'(t) = t\left(\frac{d}{dt}\sin\pi t\right) + \sin\pi t\left(\frac{d}{dt}t\right)$$

$$= t(\pi\cos\pi t) + \sin\pi t(1)$$ † Chain Rule

$$= t\pi\cos\pi t + \sin\pi t$$

$$= \pi t\cos\pi t + \sin\pi t$$

†

$$\frac{d}{dt}\sin\pi t = \frac{d}{du}\sin u \cdot \frac{d}{dt}u$$

$$= \cos(\pi t) \cdot \frac{d}{dt}\pi t$$

$$= \cos\pi t \cdot \pi$$

$$= \pi\cos\pi t$$

Or simply remember

$$\frac{d}{dt}\sin u = \cos u\frac{du}{dt}.$$

Since $u = \pi t$,

$$\frac{d}{dt}\sin(\pi t) = \cos(\pi t)\frac{d}{dt}(\pi t)$$

$$= \pi\cos\pi t.$$

32 Differentiate $F(t) = \frac{t^2}{\sqrt{t^3+1}} = \frac{t^2}{(t^3+1)^{1/2}}$

$$F'(t) = \frac{(t^3+1)^{1/2}\left(\frac{d}{dt}t^2\right) - t^2\left(\frac{d}{dt}(t^3+1)^{1/2}\right)}{\left((t^3+1)^{1/2}\right)^2}$$

$$= \frac{(t^3+1)^{1/2}(2t) - t^2\left(\frac{3}{2}t^2(t^3+1)^{-1/2}\right)}{t^3+1}$$

$$\boxed{= \frac{2t(t^3+1)^{1/2} - \frac{3}{2}t^4(t^3+1)^{-1/2}}{t^3+1}}$$

nice simplified form

$$= \frac{2t(t^3+1) - \frac{3}{2}t^4}{(t^3+1)(t^3+1)^{1/2}} = \frac{4t(t^3+1) - 3t^4}{2(t^3+1)^{3/2}} = \frac{t^4+4t}{2(t^3+1)^{3/2}} = \frac{t(t^3+4)}{2(t^3+1)^{3/2}}$$

$$\frac{d}{dt}(t^3+1)^{1/2} = \frac{d}{du}u^{1/2} \cdot \frac{d}{dt}u \qquad u = t^3+1$$

$$= \frac{1}{2}u^{-1/2} \cdot \frac{d}{dt}u$$

Or simply using $\boxed{4}$ on p200

$$\frac{d}{dt}(t^3+1)^{1/2}$$

$$= \frac{1}{2}(t^3+1)^{-1/2}\frac{d}{dt}(t^3+1)$$

$$= \frac{1}{2}(t^3+1)^{-1/2}(3t^2)$$

$$= \frac{1}{2}(t^3+1)^{-1/2} \cdot \frac{d}{dt}(t^3+1)$$

$$= \frac{1}{2}(t^3+1)^{-1/2} \cdot (3t^2)$$

$$= \frac{3}{2}t^2(t^3+1)^{-1/2}$$

Alternative: $F(t) = t^2(t^3+1)^{-1/2}$. Then by the Product Rule,

$$F'(t) = t^2\left[-\frac{1}{2}(t^3+1)^{-3/2} \cdot 3t^2\right] + 2t(t^3+1)^{-1/2}$$

$$\Rightarrow F'(t) = -\frac{3}{2}t^4(t^3+1)^{-3/2} + 2t(t^3+1)^{-1/2}.$$

36. Differentiate $y = x^2 e^{-1/x}$

By the Product Rule,

$$y' = x^2\left(\frac{d}{dx} e^{-1/x}\right) + e^{-1/x}\left(\frac{d}{dx} x^2\right)$$

$$= x^2\left(\frac{1}{x^2} e^{-1/x}\right) + e^{-1/x}(2x) \qquad ^{\dagger}\text{Chain Rule}$$

$$= e^{-1/x} + 2xe^{-1/x}$$

$$= \boxed{e^{-1/x}(1+2x)}$$

$u = -1/x$

$$\frac{d}{dx} e^{-1/x} = \frac{d}{du} e^{u} \cdot \frac{d}{dx} u$$

$$= e^{u} \cdot \frac{d}{dx} u$$

$$= e^{-1/x} \cdot \frac{d}{dx}\left(\frac{-1}{x}\right)$$

$$= e^{-1/x} \cdot \frac{x\left(\frac{d}{dx}(-1)\right) - (-1)\left(\frac{d}{dx} x\right)}{x^2}$$

$$= e^{-1/x} \cdot \frac{(0+1)}{x^2}$$

$$= e^{-1/x} \cdot \frac{1}{x^2}$$

$$= \boxed{\frac{1}{x^2} e^{-1/x}}$$

†Faster: Simply remember

$$\frac{d}{dx} e^{u} = e^{u} \frac{du}{dx}, \text{ where } u = g(x)$$

Hence,

$$\frac{d}{dx} e^{-\frac{1}{x}} = e^{-\frac{1}{x}} \frac{d}{dx}\left(-\frac{1}{x}\right)$$

$$= e^{-1/x} \frac{d}{dx}\left(-x^{-1}\right)$$

$$= e^{-1/x}\left(x^{-2}\right)$$

$$= \frac{1}{x^2} e^{-1/x}.$$

60 At what point on the curve $y=\sqrt{1+2x}$ is the tangent line perpendicular to the line $6x+2y=1$?

Solving $6x+2y=1$ for y, we have

$2y=-6x+1$

$y=-3x+\frac{1}{2}$.

The slope of this line is $m=-3$.

So the ~~perpendicular~~ tangent line perpendicular to it will have ~~$\frac{1}{3}$~~ slope $\frac{1}{3}$.

$y=\sqrt{1+2x}=(1+2x)^{1/2}$

$\boxed{u=(1+2x)}$

Thus, by the generalized power rule,

$y'=\frac{d}{du}u^{1/2}\cdot\frac{d}{dx}u$

$\frac{1}{2}u^{-1/2}\cdot\frac{d}{dx}u$

$=\frac{1}{2}(1+2x)^{-1/2}\cdot\frac{d}{dx}(1+2x)$

$=\frac{1}{2}(1+2x)^{-1/2}\cdot(2)$

$=(1+2x)^{-1/2}$

$=\frac{1}{\sqrt{1+2x}}$

$y'=\frac{1}{2}(1+2x)^{-1/2}\cdot\frac{d}{dx}(1+2x)$

$=(1+2x)^{-1/2}$

$=\frac{1}{\sqrt{1+2x}}$.

Solve the equation:

$\frac{1}{3}=\frac{1}{\sqrt{1+2x}}$.

$\sqrt{1+2x}=3$

$1+2x=9$

$2x=8$

$\boxed{x=4}$ ✓

When $x=4$, then

$y=\sqrt{1+2(4)}=3$.

Thus the point is

$\boxed{(4,3)}$.