Math 122 - Quiz 1
January 14, 2019

Name Key

No calculators. Show all necessary work.

1. Differentiate $y = \tan 2x$ with respect to x. (2 points.)

$$y' = \frac{d}{dx}\tan(2x) = \sec^2(2x)\frac{d}{dx}(2x) = \boxed{2\sec^2(2x)}$$

2. Differentiate $f(x) = e^x \cos x$ with respect to x. (2 pts.)

By the product rule,

$$f'(x) = \frac{d}{dx}(e^x \cos x) = e^x \frac{d}{dx}\cos x + (\cos x)\frac{d}{dx}e^x$$
$$= -e^x \sin x + e^x \cos x = e^x(\cos x - \sin x)$$

3. Differentiate $y = \dfrac{t \sin t}{1+t}$ with respect to t. (3 pts.)

$$y' = \frac{d}{dt}\left(\frac{t\sin t}{1+t}\right) = \frac{(1+t)\frac{d}{dt}(t\sin t) - (t\sin t)\frac{d}{dt}(1+t)}{(1+t)^2}$$ (Quotient Rule)

$$= \frac{(1+t)(t\cos t + \sin t) - t\sin t}{(1+t)^2}$$ (Product rule ; $\frac{d}{dt}(1+t) = 1$)

$$= \frac{(1+t)(t\cos t) + (1+t)\sin t - t\sin t}{(1+t)^2} = \frac{(t+t^2)\cos t + \sin t}{(1+t)^2}$$

4. Find an equation of the tangent line to the curve $y = 2\cos x - \sin x$ at the point $(0, 2)$. (3 pts.)

Slope: $y' = -2\sin x - \cos x$

$$m = y'\big|_{x=0} = -2\sin 0 - \cos 0 = -1$$

The line with slope $m=-1$ through $(0,2)$ has the equation

$$y - 2 = -1(x-0) \Rightarrow y = 2 - x$$

Extra Credit. (1 pt.)
Find the derivative of $f(x) = 3x^2 + 1$ using the definition of the derivative (i.e., not using any rules but rather the difference quotient).

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{3(x+h)^2+1-(3x^2+1)}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{3x^2}+6xh+3h^2\cancel{+1}-\cancel{3x^2}-\cancel{1}}{h} = \lim_{h\to 0}\frac{h(6x+3h)}{h} = \lim_{h\to 0}(6x+3h) = \boxed{6x}$$