

11-9 Multiplication of Binomials Containing Radicals

Objective: To multiply binomials containing square-root radicals and to rationalize binomial denominators that contain square-root radicals.

Vocabulary

Conjugates If b and d are both nonnegative; then the binomials

$$a\sqrt{b} + c\sqrt{d} \quad \text{and} \quad a\sqrt{b} - c\sqrt{d}$$

are called conjugates of one another.

Example 1 Simplify $(4 + \sqrt{5})(4 - \sqrt{5})$.

Solution The pattern is $(a + b)(a - b) = a^2 - b^2$.

$$\begin{aligned} (4 + \sqrt{5})(4 - \sqrt{5}) &= 4^2 - (\sqrt{5})^2 \\ &= 16 - 5 \\ &= 11 \end{aligned}$$

Example 2 Simplify $(2 + \sqrt{3})^2$.

Solution The pattern is $(a + b)^2 = a^2 + 2ab + b^2$.

$$\begin{aligned} (2 + \sqrt{3})^2 &= 2^2 + 2(2)(\sqrt{3}) + (\sqrt{3})^2 \\ &= 4 + 4\sqrt{3} + 3 \\ &= 7 + 4\sqrt{3} \end{aligned}$$

Simplify.

- | | | |
|-------------------------------------|-------------------------------------|-----------------------------------|
| 1. $(5 + \sqrt{3})(5 - \sqrt{3})$ | 2. $(2 - \sqrt{3})(2 + \sqrt{3})$ | 3. $(3 + \sqrt{7})(3 - \sqrt{7})$ |
| 4. $(\sqrt{15} - 2)(\sqrt{15} + 2)$ | 5. $(\sqrt{11} - 5)(\sqrt{11} + 5)$ | 6. $(\sqrt{7} + 6)(\sqrt{7} - 6)$ |
| 7. $(\sqrt{3} + 5)(\sqrt{3} - 5)$ | 8. $(6 - \sqrt{5})(6 + \sqrt{5})$ | 9. $(2\sqrt{3} - 1)^2$ |
| 10. $(4\sqrt{6} + 3)^2$ | 11. $(8 + \sqrt{3})^2$ | 12. $(\sqrt{5} + 3)^2$ |

Example 3 Simplify $(3\sqrt{2} - 7\sqrt{5})^2$.

Solution The pattern is $(a - b)^2 = a^2 - 2ab + b^2$.

$$\begin{aligned} (3\sqrt{2} - 7\sqrt{5})^2 &= (3\sqrt{2})^2 - 2(3\sqrt{2})(7\sqrt{5}) + (7\sqrt{5})^2 \\ &= (3\sqrt{2})^2 - 2(3)(7)(\sqrt{2})(\sqrt{5}) + (7\sqrt{5})^2 \\ &= 9(2) - 42\sqrt{10} + 49(5) \\ &= 18 - 42\sqrt{10} + 245 \\ &= 263 - 42\sqrt{10} \end{aligned}$$

11-9 Multiplication of Binomials Containing Radicals (continued)

Simplify.

13. $(2\sqrt{5} - \sqrt{3})(2\sqrt{5} + \sqrt{3})$

14. $(3\sqrt{7} - \sqrt{5})(3\sqrt{7} + \sqrt{5})$

15. $(4\sqrt{5} - \sqrt{17})(4\sqrt{5} + \sqrt{17})$

16. $(6\sqrt{10} + 2\sqrt{6})(6\sqrt{10} - 2\sqrt{6})$

17. $(2\sqrt{3} - 3)(3\sqrt{3} + 2)$

18. $(5\sqrt{2} + 2)(3\sqrt{2} - 2)$

19. $(2\sqrt{5} - 5\sqrt{7})(3\sqrt{5} + \sqrt{7})$

20. $(6\sqrt{11} + \sqrt{6})(2\sqrt{11} + 3\sqrt{6})$

21. $(5\sqrt{11} - 2\sqrt{3})(4\sqrt{11} + 3\sqrt{3})$

22. $(4\sqrt{6} - 3\sqrt{2})(6\sqrt{6} - 2\sqrt{2})$

Example 4 Rationalize the denominator of $\frac{2}{3 + \sqrt{5}}$.

Solution $\frac{2}{3 + \sqrt{5}} = \frac{2}{3 + \sqrt{5}} \cdot \frac{3 - \sqrt{5}}{3 - \sqrt{5}}$ Use the conjugate of the denominator.

$$= \frac{2(3 - \sqrt{5})}{9 - (\sqrt{5})^2}$$

$$= \frac{6 - 2\sqrt{5}}{9 - 5}$$

$$= \frac{6 - 2\sqrt{5}}{4}$$

Rationalize the denominator of each fraction.

23. $\frac{1}{1 + \sqrt{3}}$

24. $\frac{1}{3 + \sqrt{2}}$

25. $\frac{2}{\sqrt{5} - 1}$

26. $\frac{1}{\sqrt{7} - 2}$

27. $\frac{2 + \sqrt{5}}{1 - \sqrt{5}}$

28. $\frac{\sqrt{5} - 1}{\sqrt{5} + 2}$

29. $\frac{6}{2\sqrt{3} - 1}$

30. $\frac{7}{3\sqrt{7} + 2}$

Mixed Review Exercises

Simplify. Assume that all variables represent positive real numbers.

1. $\sqrt{72x^6}$

2. $3\sqrt{12x} \cdot 4\sqrt{3}$

3. $4\sqrt{20} - 2\sqrt{5}$

4. $3\sqrt{27} + 5\sqrt{48}$

5. $\sqrt{\frac{2}{3}} \cdot \sqrt{\frac{3}{32}}$

6. $\sqrt{1\frac{2}{3}} \cdot \sqrt{3\frac{1}{3}}$

7. $(2 - 3k^2)^2$

8. $(3p + 5z)^2$

9. $(4ab + x)(4ab - x)$

Solve.

10. $5p - 1 = 4(p + 3)$

11. $x^2 - 11x + 28 = 0$

12. $4g^2 = 100$