Math 122 - Quiz 10
March 1, 2019

Name Key

1. If $w'(t)$ is the rate of growth of a child in pounds per year, what does $\int_5^{10} w'(t)\,dt$ represent? (2 pts.)

$\int_5^{10} w'(t)\,dt$ is the increase in the child's weight in pounds from age 5 to 10.

2. Evaluate $\int_{\pi/6}^{\pi/2} \csc t \cot t\,dt$. (3 pts.)

$$\int_{\pi/6}^{\pi/2} \csc t \cot t\,dt = -\csc t \Big|_{\pi/6}^{\pi/2} = -\csc \tfrac{\pi}{2} + \csc \tfrac{\pi}{6}$$

$$= \frac{1}{\sin \pi/6} - \frac{1}{\sin \pi/2} = \frac{1}{1/2} - \frac{1}{1} = \boxed{1}$$

3. Find the derivative of the function $y = \int_x^{\sin x} \ln(1+2u)\,du$. (2 pts.)

$$\frac{dy}{dx} = \frac{d}{dx}\int_x^{\sin x} \ln(1+2u)\,du = \frac{d}{dx}\left[\int_x^0 \ln(1+2u)\,du + \int_0^{\sin x} \ln(1+2u)\,du\right]$$

$$= -\frac{d}{dx}\int_0^x \ln(1+2u)\,du + \frac{d}{dx}\int_0^{\sin x} \ln(1+2u)\,du$$

$$= -\ln(1+2x) + \ln(1+2\sin x)\frac{d}{dx}\sin x = \cos(x)\ln(1+2\sin x) - \ln(1+2x)$$

4. The velocity function (in meters per second) of a particle moving along a line is $v(t) = t^2 - 2t - 3$. (3 pts.) Find the displacement of the particle during the time interval $2 \le t \le 4$].

Displacement or net change in position $= \int_2^4 (t^2 - 2t - 3)\,dt$

$$= \left[\frac{t^3}{3} - t^2 - 3t\right]_2^4 = \left(\frac{64}{3} - 16 - 12\right) - \left(\frac{8}{3} - 4 - 6\right)$$

$$= \frac{56}{3} - 28 + 10 = \frac{56}{3} - 18 = \frac{56 - 54}{3} = \frac{2}{3} \text{ meter}$$

Extra Credit. (2 pt.) In Problem 4 find the distance traveled by the particle.

$t^2 - 2t - 3 = (t-3)(t+1) = 0 \Rightarrow t = 3, t = -1$. So $v(t) < 0$ for $2 \le t < 3$ & $v(t) > 0$ for $3 < t \le 4$.

$$\text{distance} = -\int_2^3 (t^2 - 2t - 3)\,dt + \int_3^4 (t^2 - 2t - 3)\,dt = -\left[\frac{t^3}{3} - t^2 - 3t\right]_2^3 + \left[\frac{t^3}{3} - t^2 - 3t\right]_3^4$$

$$= (-9 + 9 + 9) + \left(\frac{8}{3} - 4 - 6\right) + \left(\frac{64}{3} - 16 - 12\right) - (9 - 9 - 9)$$

$$= 4 \text{ meters}$$