

10 Inequalities

10-1 Order of Real Numbers

Objective: To review the concept of order and to graph inequalities in one variable.

Vocabulary

Inequality A statement formed by placing an inequality symbol between numerical or variable expressions.

Solutions of an inequality The values of the domain of the variable for which the inequality is true.

Solution set of an inequality The set of all solutions of the inequality.

Graph of an inequality The graph of the numbers in the solution set of an inequality.

Symbols

Inequality symbols Symbols used to show the order of two real numbers:

$$\begin{array}{ll} x > 2 & (x \text{ is greater than } 2.) \\ x < 2 & (x \text{ is less than } 2.) \\ -3 < x < 1 & (x \text{ is greater than } -3 \text{ and less than } 1.) \end{array} \quad \begin{array}{ll} x \geq 2 & (x \text{ is greater than or equal to } 2.) \\ x \leq 2 & (x \text{ is less than or equal to } 2.) \end{array}$$

Example 1 Translate the statements into symbols.

a. -2 is greater than -6

b. x is less than or equal to 5 .

Solution a. $-2 > -6$

b. $x \leq 5$

Translate the statements into symbols.

1. -2 is less than 5 .
2. -3 is greater than -4 .
3. -6 is less than or equal to -2 .
4. 4 is greater than or equal to 1 .
5. 4 is greater than 1 and less than 5.5 .
6. 0 is greater than -2 and less than 3 .
7. -5 is between 1 and -7 .
8. 3 is between -5 and 5 .
9. 3.5 is greater than 3 and 3 is greater than 0 .
10. -3.5 is less than -3 and -3 is less than 0 .
11. The number n is greater than 6 .
12. The number n is less than 12 .

Example 2 Classify each statement as true or false.

a. $-2 < 2 < 4$ b. $-1 < 5 < 3$

Solution a. $-2 < 2 < 4$ is true since *both* $-2 < 2$ and $2 < 4$ are true.

b. $-1 < 5 < 3$ is false because $-1 < 5$ is true *but* $5 < 3$ is false.

10-1 Order of Real Numbers (continued)**Example 3** Classify each statement as true or false.

a. $5 \leq 8$ b. $2 \leq 2$

Solution a. For $5 \leq 8$ to be true, *either* $5 < 8$ or $5 = 8$ must be true.

$5 \leq 8$ is true since $5 < 8$.

b. $2 \leq 2$ is true since $2 = 2$ is true.

Classify each statement as true or false.

13. $-5 < 1 < 6$

14. $-8 < 2 < 5$

15. $3 > 0 > 1$

16. $-3 < -2 < 3$

17. $8 \geq 4$

18. $13 \leq 22$

19. $-9 \leq -16$

20. $-2 \geq -3$

21. $|-2| \geq -2$

22. $|-2| \leq 0$

23. $|0.5| < -0.3$

24. $|-5| \leq |-10|$

Example 4 Solve $y + 2 \leq 3$ if $y \in \{-2, -1, 0, 1, 2, 3\}$.**Solution** Find all the values in the domain that make the inequality true.Replace y with each of its values in turn:

$-2 + 2 \leq 3$ **True**

$-1 + 2 \leq 3$ **True**

$0 + 2 \leq 3$ **True**

$1 + 2 \leq 3$ **True**

$2 + 2 \leq 3$ **False**

$3 + 2 \leq 3$ **False**

The solution set is $\{-2, -1, 0, 1\}$ Solve each inequality if $x \in \{-3, -2, -1, 0, 1, 2, 3\}$.

25. $2x < 4$

26. $3x < 3$

27. $-2x \leq 6$

28. $x + 1 < 3$

29. $-2 + x \leq 0$

30. $1 - x \geq 0$

31. $x^2 \geq 8$

32. $x^2 \leq 9$

Mixed Review Exercises

Solve.

1. $x - 4 = 11$

2. $12 = 3(c - 1)$

3. $3 - 2a = 15$

4. $\frac{x}{2} = -15$

5. $\frac{24}{y} = \frac{8}{3}$

6. $\frac{2x + 4}{4} = \frac{x + 8}{3}$

7. $3(4 + n) = 2(n - 5)$

8. $(x + 2)(x + 5) = (x + 3)^2$

9. $\frac{n}{3} + 6 = n$

10. $\frac{x}{3} = \frac{x - 5}{4}$

11. $\frac{3x}{8} + \frac{x}{4} = \frac{5}{4}$

12. $\frac{1}{2}(x + 6) = 8$