Cameron Tinker
2/21/19
Math 122

Quiz 8

pg. 389 24) Use the definition of the integral given in Theorem 4 to evaluate the integral

$$\int_0^2 (2x - x^3)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \quad \text{where } \Delta x = \frac{b-a}{n},\ x_i = a + i\Delta x$$

$$\Delta x = \frac{2}{n},\quad x_i = \frac{2i}{n},\quad f(x_i) = \left[2\left(\frac{2i}{n}\right) - \left(\frac{2i}{n}\right)^3\right] = \left[\frac{4i}{n} - \frac{8i^3}{n^3}\right]$$

$$\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}\left[\frac{4i}{n} - \frac{8i^3}{n^3}\right]\left(\frac{2}{n}\right) = \frac{8}{n^2}\sum_{i=1}^{n} i - \frac{16}{n^4}\sum_{i=1}^{n} i^3 = \frac{\overset{4}{\cancel{8}}}{n^{\cancel{2}}}\cdot\frac{\cancel{n}(n+1)}{\cancel{2}} - \frac{16}{n^4}\cdot\left[\frac{n(n+1)}{2}\right]^2$$

$$= \frac{4(n+1)}{n} - \frac{4(n+1)^2}{n^2} = 4\left(1+\frac{1}{n}\right) - 4\left(1+\frac{1}{n}\right)^2 \qquad \text{nice}$$

$$\therefore \int_0^2 (2x - x^3)\,dx = \lim_{n\to\infty}\left[4\left(1+\frac{1}{n}\right) - 4\left(1+\frac{1}{n}\right)^2\right] = \left[4(1) - 4(1)^2\right] = \boxed{0}$$

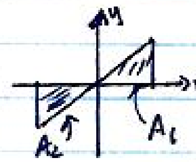
pg. 390 38) Evaluate the integral by interpreting it in terms of areas.

$$\int_{-5}^{5}\left(x - \sqrt{25-x^2}\right)dx = \int_{-5}^{5} x\,dx - \int_{-5}^{5}\sqrt{25-x^2}\,dx$$

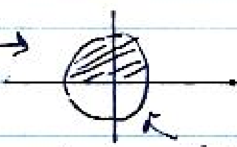
Graph of $y = x$ is a straight line

Graph of $y^2 = 25 - x^2$ is a semi-circle w/ $r = 5$.

$$\int_{-5}^{5} x\,dx = A_1 - A_2 = \tfrac{1}{2}(5\cdot 5) - \tfrac{1}{2}(5\cdot 5) = 0$$



$$\int_{-5}^{5}\sqrt{25-x^2}\,dx = \frac{\pi r^2}{2} = \frac{\pi(5)^2}{2} = 12.5\pi$$



$$\therefore \int_{-5}^{5}\left(x - \sqrt{25-x^2}\right)dx = 0 - 12.5\pi = \boxed{-12.5\pi}$$

42) Given that $\int_0^{\pi} \sin^4 x\,dx = \frac{3}{8}\pi$, what is $\int_{\pi}^{0} \sin^4\theta\,d\theta$?

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx \qquad \textbf{Here, } \int_{\pi}^{0} \sin^4\theta\,d\theta = -\int_0^{\pi} \sin^4\theta\,d\theta = -\frac{3\pi}{8}$$

$$\therefore \int_{\pi}^{0} \sin^4\theta\,d\theta = \boxed{-\frac{3}{8}\pi}$$

48) If $\int_2^8 f(x)\,dx = 7.3$ and $\int_2^4 f(x)\,dx = 5.9$, find $\int_4^8 f(x)\,dx$

$$\int_2^4 f(x)\,dx + \int_4^8 f(x)\,dx = \int_2^8 f(x)\,dx$$

$$\int_4^8 f(x)\,dx = \int_2^8 f(x)\,dx - \int_2^4 f(x)\,dx = 7.3 - 5.9 = \boxed{1.4}$$

60) Use Property 8 to estimate the value of the integral

$$\int_0^3 \frac{1}{x+4}\,dx \qquad \text{Property 8: } m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

$$(b-a) = 3 - 0 = 3$$

$$m = \frac{1}{3+4} = \frac{1}{7}$$

$$M = \frac{1}{0+4} = \frac{1}{4}$$

$$\therefore \frac{1}{7}(3) \le \int_0^3 \frac{1}{x+4}\,dx \le \frac{1}{4}(3) \qquad \boxed{\frac{3}{7} \le \int_0^3 \frac{1}{x+4}\,dx \le \frac{3}{4}}$$