Quiz 16

#13 p489 Evaluate $\int \sqrt{\cos\theta}\,\sin^3\theta\,d\theta$.

Soln. Let $u = \cos\theta$. Then $du = -\sin\theta\,d\theta$.

$$\int \sqrt{\cos\theta}\,\sin^3\theta\,d\theta = \int (\cos\theta)^{1/2}\sin^2\theta\,\sin\theta\,d\theta = \int (\cos\theta)^{1/2}(1-\cos^2\theta)\sin\theta\,d\theta$$

$$= -\int u^{1/2}(1-u^2)\,du = -\int (u^{1/2} - u^{5/2})\,du$$

$$= -\frac{2}{3}u^{3/2} + \frac{2}{7}u^{7/2} + C.$$

$$\therefore \int \sqrt{\cos\theta}\,\sin^3\theta\,d\theta = \frac{2}{7}(\cos\theta)^{7/2} - \frac{2}{3}(\cos\theta)^{3/2} + C$$

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14 p489 Evaluate $\int \frac{\sin^2(1/t)}{t^2}\,dt =: I$

Soln. Let $u = \frac{1}{t}$. Then $du = -\frac{1}{t^2}\,dt$.

$$I = -\int \sin^2 u\,du = -\int \frac{1-\cos(2u)}{2}\,du = -\frac{1}{2}\int du + \frac{1}{2}\int \cos(2u)\,du$$

$$= -\frac{1}{2}u + \frac{1}{4}\sin(2u) + C.$$

$$\therefore \int \frac{\sin^2(1/t)}{t^2}\,dt = \frac{1}{4}\sin\left(\frac{2}{t}\right) - \frac{1}{2t} + C$$

#24 p 484 Evaluate $\int (\tan^2 x + \tan^4 x)\, dx$.

Soln. $\tan^2 x + \tan^4 x = \tan^2 x\,(1 + \tan^2 x) = \tan^2 x \sec^2 x$

Thus, $\int (\tan^2 x + \tan^4 x)\, dx = \int \tan^2 x \underbrace{\sec^2 x\, dx}_{\text{Suggests } u = \tan x}$

Let $u = \tan x$. Then $du = \sec^2 x\, dx$.

So, $\int (\tan^2 x + \tan^4 x)\, dx = \int u^2\, du = \frac{u^3}{3} + C = \frac{1}{3}\tan^3 x + C$

#26 p 484 $\int_0^{\pi/4} \sec^6\theta \tan^6\theta\, d\theta =: I$

Soln. $I = \int_0^{\pi/4} \tan^6\theta \sec^4\theta \underbrace{\sec^2\theta\, d\theta}_{\text{Suggests } u = \tan\theta} = \int_0^{\pi/4} \tan^6\theta\,(\tan^2\theta + 1)^2 \sec^2\theta\, d\theta$

Let $u = \tan\theta$. Then $du = \sec^2\theta\, d\theta$. $\theta = 0 \Rightarrow u = 0$; $\theta = \frac{\pi}{4} \Rightarrow u = 1$

$$I = \int_0^1 u^6 (u^2+1)^2\, du = \int_0^1 u^6 (u^4 + 2u^2 + 1)\, du$$

$$= \int_0^1 (u^{10} + 2u^8 + u^6)\, du = \left[\frac{u^{11}}{11} + 2\cdot\frac{u^9}{9} + \frac{u^7}{7}\right]_0^1$$

$$= \frac{1}{11} + \frac{2}{9} + \frac{1}{7} = \frac{1}{11} + \frac{14+9}{63}$$

$$= \frac{1}{11} + \frac{23}{63} = \frac{63 + 256}{693} = \frac{316}{693}$$

$\boxed{28\ p\ 484}$ Evaluate $\int \tan^5 x \sec^3 x \, dx =: J$

Soln.

$$J = \int \tan^4 x \sec^2 x \underbrace{\sec x \tan x \, dx}_{u = \sec x} = \int (1-\sec^2 x)^2 \sec^2 x \sec x \tan x \, dx.$$

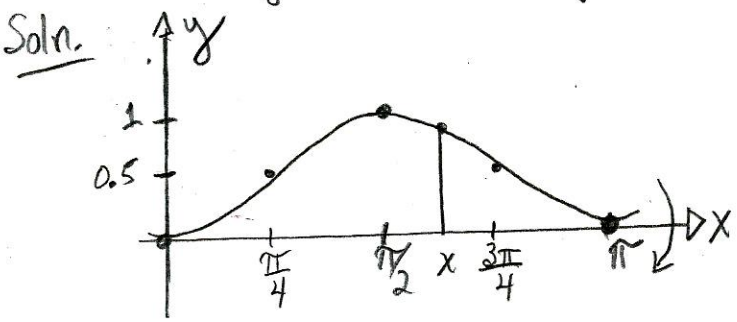
Let $u = \sec x$. Then $du = \sec x \tan x \, dx$.

$$J = \int (1-u^2)^2 u^2 \, du = \int (u^2 - 2u^4 + u^6)\, du = \frac{1}{3}u^3 - \frac{2}{5}u^5 + \frac{1}{7}u^7 + C$$

$$\therefore \int \tan^5 x \sec^3 x \, dx = \frac{1}{3}\sec^3 x - \frac{2}{5}\sec^5 x + \frac{1}{7}\sec^7 x + C$$

$\boxed{62\ p485}$ Find the volume obtained by rotating the region bounded by the curves $y = \sin^2 x$ and $y = 0$ for $0 \le x \le \pi$ about the x-axis.

Soln.



$$\sin^2\frac{\pi}{4} = \left(\sin\frac{\pi}{4}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2 = 1/2$$

$$A(x) = \pi y^2 = \pi(\sin^2 x)^2 = \pi \sin^4 x$$

$$V = \int_0^\pi A(x)\,dx = \pi\int_0^\pi \sin^4 x\,dx = \pi\int_0^\pi (\sin^2 x)^2\,dx = \pi\int_0^\pi \left(\frac{1-\cos(2x)}{2}\right)^2 dx$$

$$= \frac{\pi}{4}\int_0^\pi \left(1 - 2\cos(2x) + \cos^2(2x)\right) dx = \frac{\pi}{4}\Big[x - \sin(2x)\Big]_0^\pi + \frac{\pi}{4}\int_0^\pi \cos^2(2x)\,dx$$

$$= \frac{\pi}{4}\left[\pi - 0\right] + \frac{\pi}{4}\int_0^\pi \frac{1+\cos(4x)}{2}\,dx$$

$$= \frac{\pi^2}{4} + \frac{\pi}{4}\left[\frac{1}{2}x + \frac{1}{8}\sin(4x)\right]_0^\pi = \frac{\pi^2}{4} + \frac{\pi}{4}\left[\frac{\pi}{2}\right] = \pi^2\left(\frac{1}{4} + \frac{1}{8}\right)$$

$$= \frac{3}{8}\pi^2$$ cubic units of volume