

10-3 Solving Problems Involving Inequalities

Objective: To solve problems involving inequalities.

Example 1 The sum of two consecutive integers is less than 80. Find the pair of such integers with the greatest sum.

Solution

Step 1 The problem asks for the largest pair of consecutive integers whose sum is less than 80.

Step 2 Let n = the smaller of the two consecutive integers.
Then $n + 1$ = the larger of the two consecutive integers.

Step 3 Use the variables to write an inequality based on the given information.
The sum of two consecutive integers is less than 80:

$$n + (n + 1) < 80$$

Step 4 Solve the open sentence: $n + n + 1 < 80$

$$2n + 1 < 80$$

$$2n < 79$$

$$n < 39\frac{1}{2}$$

The largest integer less than $39\frac{1}{2}$ is 39. Thus, $n = 39$ and $n + 1 = 40$.

Step 5 **Check:** Is the sum $39 + 40$ less than 80?

$$39 + 40 \stackrel{?}{<} 80$$

$$79 < 80 \checkmark$$

39 and 40 form the largest pair of consecutive integers whose sum is less than 80.

For each of the following:

- Choose a variable to represent the number in bold face type.
- Use the variable to write an inequality based on the given information.
(Do not solve.)

- Harry, who is not yet 16 years old, is three years younger than Lena.
(Lena's age)
- After driving 125 miles, Barry still has more than 75 miles to travel.
(the total number of miles Barry will drive)

Example 2 Translate each phrase into mathematical terms.

a. The age of the house *is at least* 75 years

b. The distance *is no less than* 250 km

c. The price of the ticket *is at most* \$190

d. Her driving time to school *is no more than* 30 min

Solution

a. $a \geq 75$

b. $d \geq 250$

c. $p \leq 190$

d. $t \leq 30$

10-3 Solving Problems Involving Inequalities (continued)

For each of the following:

- a. Choose a variable to represent the number in bold face type.
 - b. Use the variable to write an inequality based on the given information.
(Do not solve.)
3. Katrina's balance in her checking account is \$160. She must deposit at least enough money in her account to be able to pay her car payment of \$295.
(the amount of deposit)
 4. Dan bicycled 12 more kilometers than one third **the number of kilometers Manuel bicycled**. Dan bicycled at most 24 km.
 5. The length of a rectangle is 7 cm longer than **the width**. The perimeter is no more than 38 cm.
 6. The sum of two consecutive odd integers is at most 185.
(the greater integer)
 7. The product of two consecutive integers is no less than 75.
(the smaller integer)

Solve.

8. The sum of two consecutive integers is less than 100. Find the pair of integers with the greatest sum.
9. The sum of two consecutive even integers is at most 180. Find the pair of integers with the greatest sum.
10. After selling 160 copies of the program to a school play, an usher had fewer than 40 copies left. How many copies of the program did the usher have originally?
11. A house and a lot together cost more than \$86,000. The house costs \$2000 more than six times the cost of the lot. How much does the lot cost by itself?
12. Andrew's salary is \$1200 a month plus a 4% commission on all his sales. What must the amount of his sales be to earn at least \$1600 each month?

Mixed Review Exercises

Solve.

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|----------------|----------------------|-------------------|
| 1. $ x = 5$ | 2. $ 1 - 5 = k$ | 3. $ y - 2 = 6$ |
| 4. $2 b = 16$ | 5. $x = -1 - (-3) $ | 6. $n = - 5 - 8 $ |

Factor completely.

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|---------------------|-------------------------|------------------|
| 7. $x^2 + 12x + 35$ | 8. $x^3 - 3x^2 - 18x$ | 9. $36x^2 - 25$ |
| 10. $2y^2 - 5y - 3$ | 11. $x^2 + 8xy + 16y^2$ | 12. $12x^3 - 3x$ |