

25 p 377

Determine a region whose area is equal to the limit

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi}{4n} \tan \frac{i\pi}{4n}.$$

Do not evaluate the limit.

Soln.

Note the sum is a Riemann sum of the form

$$\sum_{i=1}^n f(x_i) \Delta x,$$

where  $\Delta x = \frac{\pi}{4n}$ ,  $f(x) = \tan x$ , and  $x_i = \frac{i\pi}{4n} = i \Delta x$

We see

$$x_0 = 0, x_1 = \frac{\pi}{4n}, \dots, x_n = \frac{n\pi}{4n} = \frac{\pi}{4}.$$

So the interval is

$$x_0 = 0 \quad \frac{\pi}{4n} \quad \frac{2\pi}{4n} = x_2 \quad \dots \quad \frac{\pi}{4} = x_n$$

The region is bounded above by the curve  $y = \tan x$ , below by the  $x$ -axis, on the left by the  $y$ -axis, and on the right by the line  $x = \frac{\pi}{4}$ .

