Quiz 11

14 p102 Evaluate $\lim\limits_{x\to 4} \dfrac{x^2+3x}{x^2-x-12}$ if it exists.

Solution. Note that the Quotient Law cannot be used since

$$\lim_{x\to 4}(x^2-x-12) = 4^2-4-12 = 0.$$

Factoring, we have

$$\lim_{x\to 4} \frac{x^2+3x}{x^2-x-12} = \lim_{x\to 4} \frac{x(x+3)}{(x-4)(x+3)} = \lim_{x\to 4} \frac{x}{x-4}.$$

Since $\lim\limits_{x\to 4^+} \dfrac{x}{x-4} = \infty$ and $\lim\limits_{x\to 4^-} \dfrac{x}{x-4} = -\infty$, we conclude

$\lim\limits_{x\to 4} \dfrac{x^2+3x}{x^2-x-12}$ does not exist.

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16 p102 Evaluate $\lim\limits_{x\to -1} \dfrac{2x^2+3x+1}{x^2-2x-3}$ if it exists

Soln. The Quotient Rule does not apply since $\lim\limits_{x\to -1}(x^2-2x-3) = 0$.

So we try factoring:

$$\lim_{x\to -1} \frac{2x^2+3x+1}{x^2-2x-3} = \lim_{x\to -1} \frac{(2x+1)(x+1)}{(x-3)(x+1)}$$

$$= \lim_{x\to -1} \frac{2x+1}{x-3}$$

$$= \frac{2(-1)+1}{-1-3} \quad \text{(Quotient Law)}$$

$$= \frac{-1}{-4}$$

$$= \boxed{\frac{1}{4}}$$

$\boxed{17\ p102}$ Evaluate $\lim_{h\to 0} \frac{(-5+h)^2-25}{h}$ if it exists.

Soln.

$$\lim_{h\to 0} \frac{(-5+h)^2-25}{h} = \lim_{h\to 0} \frac{25-10h+h^2-25}{h} = \lim_{h\to 0} \frac{-10h+h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(-10+h)}{h} = \lim_{h\to 0} (-10+h) = \boxed{-10}$$

$\boxed{23\ p103}$ Evaluate $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3}$ if it exists

Soln.

$$\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3} = \lim_{x\to 3} \frac{\left(\frac{1}{x}-\frac{1}{3}\right)\cdot 3x}{(x-3)\cdot 3x} = \lim_{x\to 3} \frac{3-x}{3x(x-3)}$$

$$= \lim_{x\to 3} \frac{-(x-3)}{3x(x-3)} = \lim_{x\to 3} \frac{-1}{3x} = -\frac{1}{9}$$

$\boxed{24\ p103}$ Evaluate $\lim_{h\to 0} \frac{(3+h)^{-1}-3^{-1}}{h}$ if it exists.

Solution.

$$\lim_{h\to 0} \frac{(3+h)^{-1}-3^{-1}}{h} = \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{3+h}-\frac{1}{3}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{3-(3+h)}{3(3+h)}\right]$$

$$= \lim_{h\to 0} \frac{-h}{3h(3+h)}$$

$$= \lim_{h\to 0} \frac{-1}{3(3+h)}$$

$$= \frac{-1}{3(3+0)}$$ Direct Subst. Property for rational functions

$$= -\frac{1}{9}$$