

Label each of the following derivatives appropriately. And use the variables that are given. For example, differentiate  $z = t^2$ . The answer is  $z' = 2t$  or  $\frac{dz}{dt} = 2t$ .

1. Differentiate  $f(x) = 5.2x + 2.3$  with respect to  $x$ . (2 pts.)

$$\begin{aligned} f'(x) &= \frac{d}{dx} (5.2x + 2.3) \\ &= 5.2 \frac{d}{dx} x + \frac{d}{dx} (2.3) \\ &= 5.2(1) + 0 \\ &= \boxed{5.2} \end{aligned}$$

2. Differentiate  $R(t) = (3t + 1)^2$  with respect to  $t$ . (2 pts.)

So

$$\begin{aligned} R(t) &= (3t + 1)^2 = 9t^2 + 6t + 1. \\ R'(t) &= 9 \frac{d}{dt} t^2 + 6 \frac{d}{dt} t + \frac{d}{dt} 1 \\ &= 9(2t) + 6(1) + 0 \\ &= \boxed{18t + 6} \end{aligned}$$

Remark. Memorize that

$$(a \pm b)^2 = a^2 \pm 2ab + b^2.$$

So,

$$\begin{aligned} (3t + 1)^2 &= (3t)^2 + 2(3t)(1) + 1^2 \\ &= 9t^2 + 6t + 1. \end{aligned}$$

3. Differentiate  $S(p) = \sqrt{p} - p$  with respect to  $p$ . (2 pts.)

$$\begin{aligned} S'(p) &= \frac{d}{dp} \sqrt{p} - \frac{d}{dp} p \\ &= \frac{d}{dp} p^{\frac{1}{2}} - 1 \\ &= \frac{1}{2} p^{-\frac{1}{2}} - 1 \\ &= \frac{1}{2} \left( \frac{1}{\sqrt{p}} \right) - 1 \\ &= \boxed{\frac{1}{2\sqrt{p}} - 1} \end{aligned}$$

4. Differentiate  $y = \frac{x^2 + 4x + 3}{\sqrt{x}}$  with respect to  $x$ .

(2 pts.)

$$y = x^{-\frac{1}{2}}(x^2 + 4x + 3) = x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$$

$$\text{So, } \frac{dy}{dx} = \left( \frac{3}{2}x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}} \right)$$

$$= \frac{3}{2}\sqrt{x} + \frac{2}{\sqrt{x}} - \frac{3}{2(\sqrt{x})^3}$$

5. The position of a particle is  $s = t^3 - 4t$ , where  $s$  is in centimeters and  $t$  is in seconds. Find the acceleration of the particle after 5 seconds. Include units in your answer.

(2 pts.)

$$\text{Velocity: } v = \frac{ds}{dt} = \frac{d}{dt}(t^3 - 4t) = 3t^2 - 4$$

$$\text{Acceleration: } a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 4) = 6t$$

When  $t = 5$  seconds,

$$a = \left. \frac{dv}{dt} \right|_{t=5} = 6(5) = \left( 30 \frac{\text{cm}}{\text{s}^2} \right)$$

or (30 centimeters per sec per sec)

Extra credit.

(1 pt.)

Give an example of a function  $f(x)$  that is continuous at  $x = 0$  but not differentiable at  $x = 0$ .

$$f(x) = |x|$$