Quiz 15

$\boxed{8\ p463}$ Find the average value of the function $h(u) = \frac{\ln u}{u}$ on $[1,5]$.

Soln. $h_{avg} = \frac{1}{b-a}\int_a^b h(u)\,du$ where $a=1$ and $b=5$.

So, $h_{avg} = \frac{1}{4}\int_1^5 \frac{\ln u}{u}\,du$

(Let $x = \ln u$. Then $dx = \frac{1}{u}du$.
when $u=1$, then $x = \ln 1 = 0$.
when $x = 5$, then $x = \ln 5$.)

$$= \frac{1}{4}\int_0^{\ln 5} x\,dx = \frac{1}{8}x^2\Big|_0^{\ln 5} = \boxed{\frac{1}{8}(\ln 5)^2}$$

OR, $\int \frac{\ln u}{u}\,du = \int x\,dx = \frac{1}{2}x^2 + C = \frac{1}{2}(\ln u)^2 + C$.

Thus $\int_1^5 \frac{\ln u}{u}\,du = \frac{1}{2}(\ln u)^2\Big|_1^5 = \frac{1}{2}\left[(\ln 5)^2 - (\ln 1)^2\right]$

$$= \frac{1}{2}(\ln 5)^2$$

And so, $h_{avg} = \frac{1}{8}(\ln 5)^2$

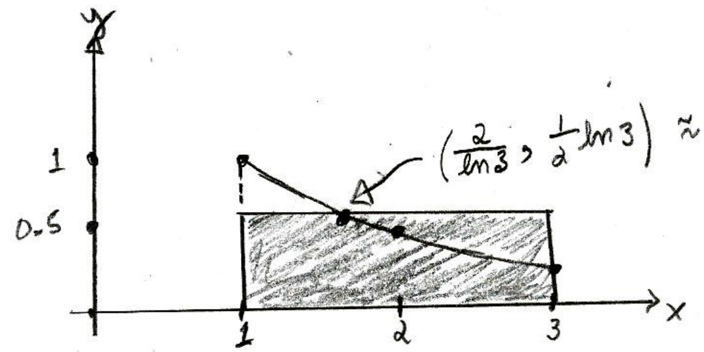
$\boxed{10\,p463}$ (a) Find the average value of $f(x) = \frac{1}{x}$ on $[1, 3]$.

<u>Soln.</u> $f_{avg} = \frac{1}{3-1}\int_1^3 \frac{1}{x}\,dx = \frac{1}{2}\left[\ln x\right]_1^3 = \frac{1}{2}\ln 3 \approx 0.549$.

(b) Find c s.t. $f(c) = f_{avg}$.

<u>Soln</u> $\frac{1}{c} = \frac{\ln 3}{2} \implies c = \frac{2}{\ln 3} \approx 1.82$.

(c) Illustrate with a graph.



The area of the above rectangle with width 2 and height $\frac{\ln 3}{2} \approx 0.549$ is equal to the area under the curve $y = \frac{1}{x}$ from $x = 1$ to $x = 3$.

18 p463 The velocity v of blood that flows in a blood vessel with radius R and length l at a distance r from the central axis is

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

where P is the pressure difference between the ends of the vessel and η is the viscosity of the blood.

(i) Find the average velocity over the interval $0 \le r \le R$.

Soln. $$V_{avg} = \frac{1}{R}\int_0^R v(r)\,dr = \frac{P}{4\eta l R}\int_0^R (R^2 - r^2)\,dr$$

$$= \frac{P}{4\eta l R}\left[R^2 r - \frac{1}{3}r^3\right]_0^R = \frac{P}{4\eta l R}\left[R^3 - \frac{1}{3}R^3\right] = \frac{P}{4\eta l R}\left[\frac{2}{3}R^3\right]$$

$$\therefore V_{avg} = \frac{PR^2}{6\eta l}$$

(ii) Compare the average velocity with the maximum velocity

Soln. From $v(r) = \frac{P}{4\eta l}(R^2 - r^2)$, we see that the velocity attains its maximum value when $r = 0$. So $V_{max} = \frac{PR^2}{4\eta l}$

Thus, $V_{avg} = \frac{2}{3}V_{max}$

2 p484 Evaluate $\int \sin^3\theta \cos^4\theta \, d\theta$.

$$\int \sin^3\theta \cos^4\theta \, d\theta = \int \sin^2\theta \, (\cos\theta)^4 \underbrace{\sin\theta \, d\theta}$$

Let $u = \cos\theta$. Then $du = -\sin\theta \, d\theta$

$$= \int (1-\cos^2\theta)\cos^4\theta \sin\theta \, d\theta$$

$$= -\int (1-u^2)u^4 \, du = \int (u^6 - u^4)\, du$$

$$= \frac{1}{7}u^7 - \frac{1}{5}u^5 + C = \boxed{\frac{1}{7}\cos^7\theta - \frac{1}{5}\cos^5\theta + C}$$

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4 p484 Evaluate the integral $\int_0^{\pi/2} \sin^5 x \, dx$.

Soln.

$$\int_0^{\pi/2} \sin^5 x \, dx = \int_0^{\pi/2} \sin^4 x \underbrace{\sin x \, dx}$$

$$\left(\begin{array}{l} \text{Let } u = \cos x. \ du = -\sin x \, dx \\ x = 0 \Rightarrow u = 1 \\ x = \pi/2 \Rightarrow u = 0 \end{array} \right)$$

$$= \int_0^{\pi/2} (\sin^2 x)^2 \sin x \, dx$$

$$= \int_0^{\pi/2} (1-\cos^2 x)^2 \sin x \, dx = -\int_1^0 (1-u^2)^2 \, du$$

$$= \int_0^1 (1-2u^2+u^4)\, du = \left[u - \frac{2}{3}u^3 + \frac{1}{5}u^5 \right]_0^1$$

$$= 1 - 2/3 + \frac{1}{5} = \frac{1}{3} + \frac{1}{5} = \frac{5+3}{15} = \frac{8}{15}$$