

2-7 Problem Solving: Consecutive Integers

Objective: To write equations to represent relationships among integers.

Vocabulary

Consecutive Integers Numbers obtained by counting by ones from any integer.

For example, $-2, -1, 0, 1,$ and 2 are consecutive integers.

Even Integer An integer that is the product of 2 and any integer.

For example, $-10, -4, 2, 6,$ and 8 are even integers.

Odd Integer An integer that is not even.

For example, $-3, -1, 7, 9,$ and 11 are odd integers.

Consecutive Even Integers Numbers obtained by counting by twos from any even integer.

For example, $-6, -4, -2, 0,$ and 2 are consecutive even integers.

Consecutive Odd Integers Numbers obtained by counting by twos from any odd integer.

For example, $-5, -3, -1, 1,$ and 3 are consecutive odd integers.

- Example 1** An integer is represented by n .
- Write the next four consecutive integers after n .
 - Write an equation to represent this relationship:
The sum of three consecutive integers starting with n is 93 .
 - Write an equation to represent this relationship:
The product of two consecutive integers starting with n is 56 .

- Solution**
- $n + 1, n + 2, n + 3, n + 4$
 - $n + (n + 1) + (n + 2) = 93$
 - $n(n + 1) = 56$

Write an equation to represent the given relationship.

- The sum of three consecutive integers is 39 .
- The sum of three consecutive integers is -51 .
- The product of two consecutive integers is 42 .
- The product of two consecutive integers is 30 .

- Example 2** Write an equation to represent this relationship:
The sum of three consecutive odd numbers is 33 .

Solution Let $n =$ the first integer, $n + 2 =$ the second integer,
and $n + 4 =$ the third integer.

$$\underbrace{\text{The sum of three consecutive odd integers}}_{n + (n + 2) + (n + 4)} \text{ is } 33. \quad \downarrow \quad = 33$$

Write an equation to represent the given relationship.

- The sum of three consecutive odd integers is 45 .
- The sum of three consecutive even integers is 72 .
- The sum of four consecutive integers is 90 .
- The sum of four consecutive even integers is -44 .

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Example 3 Write an equation to represent this relationship:
The product of two consecutive integers is 110.

Solution Let n = the first integer and $n + 1$ = the second integer.

$$\underbrace{\text{The product of the two consecutive integers}}_{n(n+1)} \text{ is } \underbrace{110}_{= 110}$$

Write an equation to represent the given relationship.

9. The product of two consecutive integers is 72.
10. The product of two consecutive even integers is 80.
11. The product of two consecutive integers is 132.
12. The product of two consecutive odd integers is 195.

Example 4 Solve over the given domain: The sum of three consecutive odd integers is 32 more than the smallest integer. What are the integers?
Domain for the smallest integer: {9, 11, 13}

Solution *Step 1* The unknowns are the three consecutive odd integers.

Step 2 Let n = the smallest integer, $n + 2$ = the middle integer, and $n + 4$ = the largest integer.

Step 3
$$\underbrace{\text{The sum}}_{n + (n + 2) + (n + 4)} \text{ is } \underbrace{32 \text{ more than the smallest integer.}}_{n + 32}$$

Step 4 Replace n in turn by 9, 11, and 13.

n	$n + (n + 2) + (n + 4) = n + 32$	
9	$9 + 11 + 13 = 9 + 32$	False
11	$11 + 13 + 15 = 11 + 32$	False
13	$13 + 15 + 17 = 13 + 32$	True

Step 5 The check is left to you. The integers are 13, 15, and 17.

Solve over the given domain.

13. The sum of three consecutive even integers is 50 more than the largest integer. What are the integers? Domain for the smallest integer: {20, 22, 24}
14. The sum of three consecutive odd integers is 72 more than the smallest integer. What are the integers? Domain for the smallest integer: {29, 31, 33}

Mixed Review Exercises

1. $(30 - 3) - (43 - 20)$ 2. $-4.5 + 2.3 - 1.7$ 3. $-2 + 3c + (-4) + 7$
4. $\frac{7}{4} + \left(-\frac{10}{4}\right)$ 5. $2\frac{2}{5} + 10 + 3\frac{3}{5}$ 6. $5\left(\frac{3}{4}\right) - 4\left(\frac{1}{4}\right) + 3\left(\frac{3}{4}\right)$
7. $-(10 - x) - (x - 15)$ 8. $14a - 4a + 5a$ 9. $10 + 4y + 5 + (-3)$